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Infusing Ethnomathematics: Proficiency of Preservice Mathematics Teachers in Task Design

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Abstract: The widespread use of mathematics throughout all facets of life and its universal presence in every culture suggests a strong intense relationship between mathematics and culture. Teachers can effectively communicate mathematical concepts to students from diverse cultural backgrounds by preparing high-level mathematical tasks that are culturally relevant. In our increasingly culturally diverse society, it is crucial for teachers to possess the necessary skills to effectively interact with students from various cultural and ethnic backgrounds. For this reason, this study aimed to assess the ability of preservice mathematics teachers to reflect the relationship between culture and mathematics in the mathematical tasks they created. In this qualitative case study, 57 preservice mathematics teachers completed 14-week elective course to develop the required basis for designing ethnomathematics tasks. The preservice teachers designed ethnomathematics tasks for a total of 19 groups upon completing the course. These tasks were analyzed in three stages. During these stages, we analyzed the conformity to the principles of activity design as outlined by Yeşildere-İmre (2020), assessed the cognitive demand levels of ethnomathematics tasks using Smith and Stein's (1998) framework, and evaluated the level of integration culture and mathematics using Ethno-cognitive demands in mathematics tasks rubric developed by researchers, in that order. As a result, the study revealed that the majority of tasks meticulously adhered to established activity design principles. Cognitive demand levels across the tasks varied significantly, with those at higher levels prompting deeper analytical thinking and problem-solving skills among students. Overall, the tasks demonstrated robust potential in facilitating a comprehensive learning experience, bridging theoretical knowledge with practical application.

Keywords: *Ethnomathematics, Culturally Relevant Pedagogy, Ethnomathematics Task, Cognitive Demand Levels, Cognitively Demanding Mathematics Tasks.*

1. INTRODUCTION

Culture and mathematics have had a reciprocal relationship throughout history. With a historical lens, this relationship exemplifies the mutual enrichment of both disciplines. Küçük (2013), highlights the inseparable connection between mathematics and culture, claiming that mathematical concepts and cultural elements mutually influence each other. Burkhardt (2008) argues that mathematical concepts are cultural artifacts inherited from ancient generations,

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whereas Barton (1996) highlights the influence of diverse cultural perspectives in shaping these concepts. This recognition has led to the emergence of the concept of ethnomathematics in mathematics education (Umbara, Wahyudin & Prabawanto, 2021). The importance of everyday mathematics becomes apparent when students connect mathematics to their cultural experiences, motivating them to perceive mathematics instruction as mandatory. Gwekwerere (2016) underscores the significance of prioritizing ethnomathematics in school mathematics, whereas Bahadır (2021) states that supporting mathematics with cultural components will improve students' cultural awareness and their capacity to comprehend mathematics. Nieto (2004) highlights that students' own cultures can be crucial tool in understanding mathematics. Brandt and Chernoff (2015) argues that experiencing mathematics in all aspects of life across cultures requires incorporating ethnomathematics into the educational curriculum. This integration promotes students' social and intellectual growth while boosting their interest and motivation (Arı, Demir & Ar, 2019; Rosa & Orey, 2011). According to Gilmer (1990), ethnomathematics allows students to study their cultural mathematics. Adam, Alangui and Barton (2003) have suggested that initially, children's mathematical thinking is shaped by mathematical concepts within their own culture and subsequently developed through school mathematics. Supriadi (2019) emphasizes that teachers' proficiency in this area enables the integration of ethnomathematics into curricula. This proficiency prompts the inquiry of how these connections should be established, indicating that one of the instruments that can facilitate the observation of these connections in the teaching process is the formulation of mathematics tasks. Studies have shown that mathematics tasks are effective in the teaching process (Doyle, 1988; Estrella et al., 2020; Radmehr, 2023) and they allow students to contextualize mathematics in different contexts and relate it to other disciplines (Estrella et al., 2020; Smith & Stein, 1998; Zaslavsky, 2005). Nevertheless, it is also emphasized that mathematics tasks can impede learning, despite their capacity to facilitate higher-level thinking (Stein, Grover & Henningsen, 1996; Stein & Smith, 1998). Hence, it is crucial to prioritize the cognitive demand levels (CDLs) of mathematics tasks to successfully attain the desired goals in the teaching process. Researchers investigating the CDLs of mathematics tasks have categorized these tasks into four categories: memorization, procedure without connections, procedures with connections, and doing mathematics (Stein & Smith, 1998). Memorization tasks generally emphasize the recall of learned algorithms and definitions, whereas procedures without connections challenges necessitate the use of conventional formulas and algorithms to arrive at a solution. In procedures with connections tasks, students' responsibility is to make connections and use different forms of representation to understand the focused mathematical concepts to complete the task. These tasks require students to use higher-level thinking skills (Stein et al., 2000; Wakhata, Mutarutinya & Balimuttajjo, 2023) and the development of these skills has become a significant focus in recent studies (Bağdat, 2019; Creager, Snider & Parrish 2021; Gülbağcı-Dede, Özen-Ünal & Yılmaz, 2023; Güzel, Bozkurt & Özmantar, 2020; Jones, 2015; Matthews, Jones & Parker, 2013; Polat & Dede, 2023). Redfield and Rousseau (1991), Hacısalihoğlu Karadeniz and Şehit (2022) have highlighted the beneficial impact of higher-level cognitive tasks on students' achievements. This emphasizes the importance of choosing tasks that necessitate higher-level cognitive skills in classrooms, rather than those that only demand lower-level cognitive skills (Norton & Kastberg, 2012). The implementation of culturally relevant, cognitively demanding mathematics tasks by teachers will help students from diverse cultural backgrounds learn how to convey mathematical ideas and also understand what cultural fallacies might arise (Rosa & Orey, 2020). This can facilitate the creation of a more comprehensive and significant comprehension of mathematics for all students, irrespective of their cultural backgrounds. Indeed, Levenson (2022) emphasizes that preservice mathematics teachers (PMTs) and mathematics teachers need to consider students' cultural backgrounds when planning the teaching process. Therefore, mathematics tasks are one of the most effective ways for future mathematics teachers to integrate ethnomathematics into mathematics classrooms. Recent studies have focused on the CDLs of mathematics tasks, but studies including ethnomathematics have been limited (Jones, 2015; Yılmaz, 2022). Hence, it is important to determine the skills of PMTs in reflecting ethnomathematics in the mathematics tasks they design.

2. LITERATURE REVIEW

2.1. Ethnomathematics

Ethnomathematics serves as an alternative to mathematics education in developing countries with colonial histories, which have been shaped under Western hegemony and distant from local cultures. It is a discipline that allows individuals to investigate mathematical knowledge based on their own cultural backgrounds. During the 1970s and 1980s, educators reacted against Eurocentric approaches by suggesting alternatives to the teaching of 'imported school mathematics' (Gerdes, 1996): Indigenous mathematics, informal mathematics, spontaneous mathematics, oppressed mathematics, non-standard mathematics, hidden or frozen mathematics, and folk mathematics. This diversity was unified under the ethnomathematics approach initiated by Ubiratan D'Ambrosio in the 1970s aimed at analyzing intercultural mathematical knowledge, gaining momentum with the establishment of The International Study Group on Ethnomathematics [ISGEm] in 1985. The emergence of ethnomathematics as a research field signifies the acknowledgment of the presence of many mathematical understandings. Gerdes (1995) explains the primary characteristics of ethnomathematics, which can be categorized into four main sections: First and foremost, ethnomathematics is a comprehensive conceptual framework that incorporates a range of activities including counting, measuring, designing, playing, and explaining. Secondly, this discipline focuses on emphasizing and analyzing the impact of sociocultural elements on the acquisition, instruction, and growth of mathematical knowledge. Thirdly, it treats mathematics as a cultural product. Lastly, every culture and subculture has cultivated its own distinct mathematics, demonstrating that mathematics as a cultural product, has undergone development over time and is employed by individuals across the globe. In this context, ethnomathematics explores the historical and cultural aspects of mathematics, investigating how mathematics has been used to create cultural artifacts and how informal mathematical practices from various cultures can be integrated into academic mathematics (Akıntude et al., 2019; Weldeana, 2016). This field emphasizes a broad conceptualization of mathematics, identifying its existence through diverse mathematical practices (Umbara, Wahyudin & Prabawanto, 2021). For instance, the study of ethnomathematics aids in raising awareness of history and culture through mathematical concepts, as seen in the exploration of ethnomathematics values in cultural sites like the Temple of Heaven in Beijing (Zhang et al., 2021). For this reason, integrating ethnomathematics into mathematics education plays a significant role in extracting mathematical knowledge from students' cultural backgrounds. Ethnomathematics can be utilized as a practical teaching approach, particularly benefiting underrepresented students and enhancing their learning experiences (Çenberci & Horzum, 2023; Nur et al., 2020) and supporting awareness of real-life examples (Esen & Saralar-Aras, 2022) .

2.2. Culturally Relevant Pedagogy and Ethnomathematics

Culturally relevant pedagogy (CRP) is an educational approach that recognizes the importance of integrating students' cultural backgrounds, identities, and prior experiences into the teaching and learning process. It is rooted in the belief that learning is a socially mediated process that should connect to students' cultural and linguistic experiences (Durden et al., 2015; Yürüko et al., 2024). CRP aims not only to integrate students' values, experiences, and perspectives into the curriculum but also to empower them to achieve academic success, develop cultural competence, and cultivate critical consciousness (Esposito et al., 2012; Shultz et al., 2023). This approach utilizes locally situated aspects of culture critical to students in instructional practices (Kondo, 2022), emphasizing the connection between learning materials and students' lives. Ethnomathematics, on the other hand, focuses on the cultural aspects of mathematics, viewing it as a cultural concept embedded in the values and intelligences of different communities (Stevens, 2021). Ethnomathematics and CRP-based approaches to mathematics curriculum are designed to enhance students' understanding of mathematical content by making it more culturally relevant and connected to their lived experiences (Rosa & Orey, 2020).

The connection between CRP and ethnomathematics lies in their shared objective of making education more inclusive and meaningful for students from diverse backgrounds. PMTs play a vital role in this process, as they are in the formative stages of developing their teaching practices and beliefs. However, it is important to address that many in-service teachers are resistant to change and lack knowledge about ethnomathematical approaches and integration methods (Sunzuma & Maharaj, 2019). Therefore, enhancing PMTs' understanding and acceptance of these approaches during their formative training is crucial for future classroom implementation. This highlights the importance of addressing PMTs' beliefs and knowledge base to improve their instructional strategies. Colvin and Tobler (2013) emphasize that PMTs need to make connections between course materials and culturally relevant contexts, allowing students to draw on their cultural experiences to interpret what they learn. This builds PMTs' confidence in mathematics knowledge for teaching and enables them to deliver effective instruction in culturally relevant ways. Therefore, by being sensitive to students' cultural orientations, backgrounds, and experiences, teachers can create a student-centered learning environment that supports diverse learners (Jia & Nasri, 2019). By embracing CRP and ethnomathematics, PMTs can promote social justice, equity, and inclusivity in mathematics education, ultimately fostering a more engaging and enriching learning experience for all students.

2.3. Implementing Ethnomathematics Tasks in Mathematics Education

Mathematics tasks are complex problems, operations, or applications that are intended to direct students' attention to a particular mathematical idea (Stein, Grover & Henningsen, 1996; Stein & Smith, 1998). These tasks are essential tools that determine the efficacy of the teaching process and function as a key source for student learning (Wilhelm, 2014). Mathematics tasks have been acknowledged as one of the seven teaching standards by worldwide institutions, highlighting their significance (National Council of Teachers of Mathematics, [NCTM], 2006). Classifying mathematical tasks based on their content is crucial in this context to comprehend and improve the learning process. Various frameworks have been proposed to classify these tasks based on different criteria such as cognitive demands, characteristics, and the nature of the mathematical activity involved (Breen & O'Shea, 2019; Donoghue, 1999; Stein, Grover & Henningsen, 1996). Among these mathematics tasks are ethnomathematics tasks.

Ethnomathematics tasks are defined as culturally embedded mathematics tasks that are designed to connect mathematics and culture, aiming to bridge the gap between mathematics and culture (D'Ambrosio, 1985; Ergene et al., 2020; Presmeg, 1998). These tasks not only improve students' comprehension of mathematics and problem-solving skills but also foster cultural value and inclusivity in education (Nur et al., 2020; Presmeg, 1998; Supriyadi et al., 2023). Bahadır's (2021) study highlights how ethnomathematics inspired by everyday life can be integrated with different disciplines. This study demonstrates how cultural contexts can be merged with mathematics and how such activities can enhance students' mathematical problem-solving abilities. In this context, Bahadır elucidates how ethnomathematics tasks can enhance effective learning and mathematical understanding through the use of mathematical problem-solving strategies across cultural transitions. Brandt and Chernoff (2015) emphasize the significance of students first becoming acquainted with the mathematics of their own cultures and subsequently incorporating culture into school mathematics to foster awareness in the learning process. By incorporating ethnomathematics into the curriculum, teachers can create a more engaging and culturally relevant learning experience for students. This process facilitates students' acquisition of mathematical knowledge in a more significant manner, while also allowing them to appreciate the relevance of mathematics in their cultural contexts and daily experiences. According to Peni (2019), students can have a deeper understanding of mathematics by connecting it to their own cultures, which helps them realize that mathematics goes beyond theorems and formulae. Stein and Lane (1996) highlight the importance of tasks applied in the classroom in the process of understanding mathematics. To be successful in this process, teachers must possess a comprehensive awareness of the relationship between culture and mathematics, as well as the requisite knowledge and skills (Supriadi, 2019). Additionally, Orey and Rosa (2008) emphasize the importance of sustaining cultural integration throughout the task.

2.4. Cognitive Demand Levels (CDLs) and Cognitively Demanding Mathematics Tasks

By supporting student learning, mathematics tasks improve the efficacy of the instructional process (Stein et al., 2000). Specifically, a significant portion of time in mathematics classes is spent on performing mathematics tasks (Roth & Givvin, 2008). These tasks have the potential to either facilitate or impede students' higher-order thinking (Stein & Smith, 1998; Stylianides & Stylianides, 2008). The effectiveness of mathematics tasks varies based on their CDLs (Ayres, 2006; Estrella et al., 2020; Polat & Dede, 2023; Wakhata, Mutarutinya & Balimuttajjo 2023). Therefore, considering the CDLs of students in the designed tasks is crucial for increasing the efficiency of the instructional process. Indeed, mathematics tasks and activities that create environments allowing students to think, solve problems, make connections between concepts, and draw inferences provide experiences for students to use higher-order thinking skills (Barnett et al., 2024; Özkale & Aprea, 2023). Research suggest that the effectiveness of instructional strategies and the enhancement of learning outcomes are strongly linked to the degree of alignment between these tasks and students' cognitive abilities (Estrella et al., 2020; Ni et al., 2018; Yıldız & Güven, 2016). Stein and Lane (1996) introduced a four-level taxonomy to enable observation of the way students do mathematics in the classroom. In this way, they can use this taxonomy to categorize the levels of mathematical thinking exhibited by students in mathematics classes (Please see Appendix 1). These levels include: 1-memorization, 2-procedures without connections, 3-procedures with connections, and 4-doing mathematics. The first two levels represent low CDLs, while the other two levels represent high CDLs. This taxonomy is also used for classifying mathematics tasks (Agterberg et al., 2022; Smith & Stein, 1998; Stein & Smith, 1998). Tasks at the low CDL typically necessitate students to recall learned rules, repeat without thinking, complete procedures accurately, and avoid making connections between concepts (Stein et al., 2000).

Various studies have shown a growing focus on higher-order thinking skills in mathematics instruction in recent years. In their study, Yılmaz (2022) sought to assess the proficiency of middle school PMTs in designing ethnomathematics tasks as part of a teacher training program. The salient feature of the tasks designed by the PMTs was their adeptness in combining culture with mathematics. Furthermore, the PMTs' inclination to revise their tasks following classroom discussions was also noteworthy. While making these modifications, it was discovered that PMTs attempted to alter the CDLs of the tasks. It was discovered that just one group was incapable of modifying the CDLs of their tasks. Another study by Gülbağcı-Dede, Özen-Ünal and Yılmaz (2023) aimed to determine the CDLs of tasks recommended for use in elementary distance education. The study's findings revealed that most tasks had a low-level cognitive demand, but several tasks were discovered to contain mathematical errors. According to Simon and Tzur (2004), tasks that require high cognitive demand allow chances to come across non-routine situations, which in turn engage students with challenging and intricate mathematical concepts. According to Stein et al. (2000), tasks with high-level cognitive demands contribute to students' meaningful, deep, and enduring learning. Research suggest that the CDLs of mathematics tasks play a crucial role in determining how and to what extent students' higher-order thinking skills will be stimulated (Henningsen & Stein, 1997; Ni et al., 2018; Smith & Stein, 1998; Stein et al., 1996; Wakhata, Mutarutinya & Balimuttajjo, 2023). Indeed, The National Council of Teachers of Mathematics (NCTM, 2014) explicitly advocates for the incorporation of cognitively demanding tasks in educational settings. Ruk (2020) highlights the crucial significance of sustaining the high level of cognitive demand that was initially established in mathematics tasks throughout their implementation. Therefore, tasks that require a high level of cognitive demand might be characterized as cognitively demanding. Utilizing such tasks in mathematics education yields advantages for both students and teachers as they offer structured opportunities for engaging in advanced cognitive processes, solving problems, and employing mathematical concepts in complex scenarios (Coşkun et al., 2023; Parrish & Bryd, 2022). This could potentially increase the percentage of successful students in the mathematics courses that teachers instruct. Consequently, the significance of PMTs who are at the early stages of their professional journeys and teachers, play a central role in the design, adaptation, and implementation of mathematical tasks, is undeniable. Research show that the choices and use of mathematics tasks by teachers have a substantial impact on the cognitive demands required by their students (Boston & Smith, 2011; Hsu & Silver, 2014; Jackson et al., 2013; Ni et al., 2018; Parrish & Martin, 2022). For example, Jackson et al. (2013) and Hsu and Silver (2014) have demonstrated that tasks with high cognitive demands can improve students'' comprehension of concepts and reasoning skills. Moreover, the capacity of teachers to maintain the cognitive demands of the tasks during the implementation is vital. When teachers simplify tasks during implementation, students' capacity for learning diminishes (Boston & Smith, 2011; Parrish & Bryd, 2022). Studies also indicate that professional development initiatives that concentrate on the selection and implementation of cognitively demanding tasks improve teachers' practices. These initiatives guarantee improved student outcomes by maintaining the cognitive demands of tasks (Ni et al., 2018; Norton & Kastberg, 2012). Bağdat (2019) aimed to investigate the influence of a professional development program, utilizing the framework of five practices, on the classroom practices employed by two mathematics teachers and the CDLs of the mathematics tasks they prepared. The study revealed that the teachers failed to reach an adequate level of meticulous foresight when designing their tasks during the professional development program. Nevertheless, they primarily executed high-level tasks while maintaining their CDLs. Güzel, Bozkurt and Özmantar (2020) examined how middle school mathematics teachers perceive the objectives of tasks in instructional documents and whether these perceptions vary by the CDLs. The study found that teachers had varying perceptions of the objectives of a specific task. Therefore, it is vital for PMTs, at the beginning of their professional development, to be able to prepare cognitively demanding tasks and deeply understand their students' mathematical thinking levels. The ability to use this comprehension in practical situations is crucial for their professional growth and for improving their student's mathematical skills. Still, research indicate that PMTs encounter challenges when it comes to comprehending, organizing, and executing cognitively demanding mathematics tasks (Bağdat, 2019; Creager, Snider & Parrish, 2021; Jao & Sahmbi, 2023; Whittington & Teknumru-Kısa, 2020). Hence, PMTs must gain practical experience with mathematics tasks of this nature throughout their preservice training to prevent obstacles in effectively executing cognitively demanding tasks in their forthcoming careers (Pečiuliauskienė & Kaminskienė, 2022).

2.5. Culturally Relevant Cognitively Demanding Mathematics Tasks Framework

The Culturally Relevant Cognitively Demanding (CRCD) Mathematics Tasks Framework, has been designed as a model that integrates CRP with cognitively demanding tasks (Matthews, Jones & Parker, 2013; Stein et al., 2000). This framework was developed to address the increasing diversity in classrooms and to make mathematics accessible to every student.

Matthews, Jones and Parker (2013) have enhanced the Mathematics Task Framework, initially suggested by Stein et al. (2000), by integrating cultural contexts into mathematics classrooms. This integration includes dimensions related to relationships, culture, and social change. The expansion has led to the development of the CRCD Mathematics Tasks Framework, which consists of two main perspectives. The first perspective is based on the high cognitively demanding tasks defined by Stein et al. (2000), which encompass (a) procedures involving concepts, mathematical meaning, culture, and society, and (b) mathematics tasks aimed at intellectual, cultural, political, and social empowerment. The second perspective incorporates features of mathematics tasks from the existing literature on CRP (Gutstein et al., 1997; Ladson-Billings, 1997, 2009; Leonard & Guha, 2002; Matthews, 2003; Tate, 1995). Within this extended framework, tasks are pertinent to the real world and empower students from diverse cultural backgrounds to understand the world, openly criticize society, and make significant decisions that impact themselves, their communities, and the world (Matthews et al., 2013). Jones (2015) has concentrated on the development and implementation of culturally relevant, cognitively demanding tasks. The researcher employed the CRCD Mathematics Task Evaluation Table, established by Matthews, Jones and Parker (2013), to analyze the mathematics tasks generated by mathematics teachers. Bankston (2022) investigated the experiences of African American students engaged in culturally meaningful and cognitively demanding mathematics tasks. Additionally, they examined students' experiences with mathematics tasks that involved levels of 'doing mathematics' or 'procedure with connections.' The CRCD Mathematics Tasks Framework, when used in conjunction with the social and learning settings experienced during these activities, functions as a tool to assist teachers in choosing cognitively demanding mathematics tasks for students from diverse cultural backgrounds (Jones, 2015). Teachers can design tasks that are relevant to their students' lives and cultural backgrounds while implementing this framework (Agterberg et al., 2022). For example, teachers can generate geometric problems based on traditional architecture, fostering collaborative learning settings that promote the exchange of cultural knowledge among students, as they collaborate to tackle complicated problems. The objective is to establish a learning environment that is inclusive and equitable, accommodating students from all backgrounds, and ultimately leading to significant cognitive and academic achievements (Brantlinger, 2013; Levenson, 2022; Radmehr, 2023). The advantages of culturally relevant cognitively demanding mathematics tasks in teaching environments have also caught the attention of the National Council of Supervisors of Mathematics [NCSM] (2020), which values teachers' pedagogical development. NCSM prioritized student learning outcomes and emphasized the role of providing equitable and high-quality mathematics experiences for all students. Following this perspective, NCSM has revised the 'CRCD Mathematics Tasks' framework originally developed by Matthews, Jones and Parker (2013). Mathematics tasks defined in 'The Revised CRCD Mathematics Task Framework' are examined at three levels, each focusing on increasing cognitive difficulty and social interaction requirements. The Emerging Level encompasses tasks that necessitate students to employ several strategies and representations to attain a profound comprehension of mathematics. Furthermore, these tasks are intricately linked with other subjects, concepts, or disciplines. The Developing Level extends these tasks by prompting students to analyze and interrogate real-life scenarios within their cultural and social backgrounds. This process influences their mathematical strategies and solutions based on their particular communities and cultures. The Exemplary Level offers students opportunities to address structures, assumptions, and social injustices of their communities and the world by developing mathematical solutions to these issues. This level promotes the conscious and effective participation of students by combining mathematical thinking with the pursuit of social justice and societal change. This integrated approach not only improves students' mathematical skills but also increases their sense of social responsibility and cultural awareness.

2.6. Purpose of the Research

The objective of this research is to assess the capacity of PMTs to integrate culture and mathematics through the design of ethnomathematics tasks. Within this context, the subsequent sub-problems were examined:

1. What are the cognitive demands of the mathematics tasks created by PMTs?
2. To what extent are PMTs skilled at incorporating culture and mathematics in their created ethnomathematics tasks?

3. METHODOLOGY

3.1. Research Design

A comprehensive examination of the topic under consideration is provided through a case study methodology in the current research, which is characterized as a qualitative study. Using a case study approach involves in-depth analysis and investigation of a specific group or situation, as Merriam (2009) explained. By employing this research method, it is possible to gain a comprehensive understanding of the intricate mechanisms involved in implementing an instructional approach based on ethnomathematics tasks. It specifically focuses on the CDLs of mathematics tasks designed by PMTs and their capabilities to integrate culture with mathematics in these ethnomathematics tasks. Moreover, the study seeks to provide critical reflection on how PMTs design their ethnomathematics tasks, how they relate mathematics to culture, and the cognitive levels they aim for throughout this process.

3.2. Study Group

The study included 57 PMTs who were enrolled in the Mathematics Teaching Program at an official university in the Central Anatolia province of Türkiye during the spring semester of the 2022-2023 academic year. The PMTs enrolled in a 14-week elective course named 'Activity Design in Mathematics Teaching.' This course consisted of subjects such as the essence of mathematical activity, its evolution over time, the importance and necessity of incorporating activities in mathematics instruction, a classification of activities, tasks according to CDLs, principles for designing activities, ethnomathematics tasks, the performing of mathematics tasks in the classroom, and lastly, activity design and implementation. During the implementation step of the instruction, the PMTs were instructed to create groups of three to design ethnomathematics tasks. The current research was conducted with the participation of 19 groups on a volunteer basis. PMTs organized themselves into pairs or trios, limited to a maximum of 3 individuals, based on their closest friends and those with whom they could collaborate and communicate most effectively. During the process of constructing these groups, PMTs were instructed to build a group of people whom they could easily get along with and whom they believed could generate high-quality work. The tasks provided by each group were progressively labeled starting from the first group's task as T1, the second group's task as T2, and so on, up to the nineteenth group's task as T19.

3.3. Data Collection Tools and Data Collection Process

The data collection process was carried out digitally to include volunteers from many provinces and districts. The research data consisted of ethnomathematics tasks, which were documents created by groups of PMTs as part of their mathematics education course on activity design. In this course, the purpose and significance of utilizing activities in the teaching of mathematics, the characteristics of activities used in mathematics teaching, the matters to be considered in the preparation and implementation of activities, and the evaluation studies of sample activities have been conducted. The relationship between mathematics and culture, the mathematical thought structures of different cultures, the basic principles of research in the field of ethnomathematics, and the importance of incorporating ethnomathematics studies into classroom practices were, however, emphasized, as preservice teachers are required to design ethnomathematics activities. According to Court et al. (2017), documents offer valuable insights regarding individuals' habits, preferences, and self-representation. They provide a brief insight into culture and can be less intrusive than participant observation, promoting objectivity in data collection. PMTs were instructed to create ethnomathematics tasks within the given framework, considering the principles of activity design that they had previously learned in theory. The PMTs created their ethnomathematics tasks within this framework by following the principles of design, contents, and expectations (Yeşildere-İmre, 2020).

Throughout all ethnomathematics tasks, PMTs successfully expressed their actions and methodologies, adhering to the design criteria. These included the PMTs' consideration of the cultural aspects of ethnomathematics tasks, the mathematical expression of content, its accuracy, and its suitability for the cognitive development levels of the students. Nevertheless, no instruction was provided at the specific point in the ethnomathematics tasks when cultural elements should be included or how these elements should be integrated into the mathematical content. Ultimately, PMTs who participated in distance education were instructed to collaboratively develop ethnomathematics tasks and submit them by utilizing a link generated by researchers via Google Forms. The PMTs were initially provided with an informed consent form and a voluntary participation consent form. Subsequently, they were requested to upload their completed tasks. The obtained documents were used primarily to determine the CDLs in ethnomathematics tasks, under the research objectives. Later, the abilities of PMTs to integrate mathematics with culture in ethnomathematics tasks that require a high level of cognitive demands were investigated.

3.4. Data Analysis

This study involved a two-stage analysis of ethnomathematics tasks created by PMTs, under the research problems. Descriptive analyses were employed at both levels. The initial phase involved evaluating the ethnomathematics tasks based on Smith & Stein's (1998) framework called 'Characteristics of Mathematical Instructional Tasks'. The assessment encompassed different levels, including lower-level demands such as memorization and procedures without connections, as well as higher-level demands such as procedures with connections and engaging in mathematical problem-solving.

CDL was used to evaluate the tasks and establish the criteria they primarily fulfilled. Both researchers agreed that Task6, Task10, and Task12 were characterized by high CDLs. However, they encountered difficulty in determining whether these tasks fell under the category of 'procedures with connections' or 'doing mathematics.' The challenges arose from the complex nature of the tasks and the variety in instructional design. The inclusion of items from various cognitive levels in the tasks made the categorization process more complex. Researchers, acknowledging that ethnomathematics tasks could fit into more than one category, decided to apply Smith & Stein's (1998) framework more flexibly. Initially, a systematic evaluation of the prominent characteristics of each task was conducted, and these analyses were subsequently reviewed based on Smith & Stein's framework. Afterward, the potential impacts of the tasks on student interactions and learning outcomes were considered. This process enabled a more precise classification of the tasks and set objective criteria for determining CDLs.

Finally, the reasons why the tasks could not fit into other categories were examined. For instance, Task6, which utilized Piri Reis's World Map and daily situations and measured students' ability to explain the position of one point relative to another using direction and units, was evaluated at a high CDL. Nevertheless, there were conflicting opinions regarding the appropriate classification (procedures with connections or doing mathematics). Therefore, Task6 was reassessed, and both researchers determined which items of the CDLs the tasks met. Consequently, it was determined that Task6 mainly belonged to the category of procedures with connections. Likewise, all remaining tasks were assessed using the same approach (see Table 1).

Table 1. *An Illustrative Case of Data Analysis for Task6*

Justifications	Explanations
Justifications for Task6	The task appears to embody characteristics of both the 'procedures with connections' and 'doing mathematics' levels. However, it can be said that the 'procedures with connections' level is more predominant in this task. The task aims to engage students in understanding mathematical concepts and ideas, by implementing extensive procedures associated with fundamental conceptual ideas and helping develop by connecting multiple representations. The task necessitates cognitive effort from students and guarantees achievement by meticulously adhering to overall processes, which are unique characteristics of the 'procedures with connections' level. The complex and non-algorithmic thinking required for the 'doing mathematics' level, such as the lack of a predictable approach or solution path in the problem, discovering and understanding the nature of mathematical concepts, processes, or relationships, and self-monitoring or regulating cognitive processes, are not emphasized in this task. While the task allows students to participate in mathematical reasoning and application, its structure is better suited for the 'procedures with connections' level. The task seems to serve as a guide for students in comprehending mathematical concepts employing specific questions and problem-solving activities in a specific context. Thus, the 'procedures with connections' level can be considered more dominant for this task.

Ultimately, this approach has enhanced our comprehension of the cognitive processes associated with each task and has provided a more precise categorization of the tasks. Within this framework, it was determined that Task2 and Task4 did not adhere to the principles of activity design (Yeşildere-İmre, 2020), whereas Task11 was found to have a low-level cognitive demand based on Smith and Stein's framework (1998). All remaining ethnomathematics tasks were found to have a high-level cognitive demand. These tasks serve as the basis for the second phase of data analysis.

The second phase of data analysis focused on evaluating the proficiency of PMTs in combining culture and mathematics. This was performed through ethnomathematics tasks that were designed based on activity design principles and at the higher-level cognitive demands (excluding Task2, Task4, and Task11). It was taken advantage of the frameworks of 'CRCD Mathematics Tasks' produced by Matthews, Jones and Parker (2013) and 'The Revised CRCD Mathematics Task Framework' by NCSM (2020) in data analysis procedures. In this context, to examine the ethnomathematics tasks prepared by PMTs in groups, the researchers developed the 'Ethno-Cognitive Demands in Mathematics Tasks Rubric,' as presented in Table 2. The ethnomathematics tasks designed by PMTs were evaluated using this rubric, which was based on eight criteria and rated as high (3 points), moderate (2 points), low (1 point), and very low (0 points).

The tasks were classified into three groups based on their scores, as outlined in Table 2: '*Initial Explorations*', '*Building Bridges*', and '*Beyond Horizons*'. Tasks scoring 0-7 points on the

ECD-MTR fall into the '*Initial Explorations*' category, which encompasses the earliest stages of incorporating mathematical and cultural elements, with a focus on the possibilities for exploration. Tasks in this category acknowledge the possible connection between culture and mathematics but are receptive to growth and exploration. PMTs responsible for designing these tasks are known as '*Discovery Travelers*'. Tasks scoring 8-16 points on the ECD-MTR fall into the '*Constructing Bridges*' category, which seeks to establish connections between mathematics and everyday life, cultural contexts, and social issues. Ethno-Cognitive Demands in Mathematics Tasks in this category contribute significantly to improving cognitive and cultural richness by transitioning between different concepts and representations. PMTs responsible for designing these tasks are known as '*Bridge Builders*'. Tasks scoring 17-24 points on the ECD-MTR fall into the '*Beyond Horizons*' category, which includes tasks that thoroughly integrate mathematical and cultural content, enabling students to engage in comprehensive inquiries about themselves and the world. These tasks combine mathematical thinking with social consciousness. The tasks in this category suggest an expansion of the limits of learning and comprehension, launching a voyage toward uncharted territories. '*Cultural Architects*' is the term used to describe PMTs who developed these tasks in the beyond horizons category. The data analysis processes were conducted separately by each researcher. Afterward, the agreements and discrepancies in coding were discussed, and the required modifications were implemented. To achieve consensus, a total of five online meetings were conducted, with each meeting lasting between two and four hours.

Table 2. *Ethno-Cognitive Demands in Mathematics Tasks Rubric (ECD-MTR)*

Criteria/Point	High (3 point)	Moderate (2 point)	Low (1 point)	Very low (0 point)
1. Integrating culture and mathematics				
2. Referring to a social situation				
3. Providing daily life context				
4. Addressing the relationship between topics and concepts				
5. Using transition between representations				
6. Being mathematically rich				
7. Being cognitively demanding				
8. Being embedded in cultural activity				

3.5. Ethical Consideration

The research received approval from the Necmettin Erbakan University Social and Humanities Scientific Research Ethics Committee (decision number 2023/127) following an evaluation on 10.03.2023, with no ethical concerns identified.

4. FINDINGS

4.1. CDLs of Ethnomathematics Tasks

The CDLs of mathematics tasks designed by PMTs were initially assessed for compliance with the principles of activity design during the analysis. Two out of the 19 mathematics tasks, specifically Task2 and Task4, were found not to follow these standards and were eliminated from the evaluation. Table 3 presents the data about the CDLs of the remaining ethnomathematics tasks.

Table 3. *Explanations Regarding the Cognitive Demand Levels of the Designed Mathematics Tasks*

Task	CDLs	Explanations	Task	CDLs	Explanations
Task1	DM	M1,M2,M3,M6	**Task12**	PwithC	İ1,İ2
Task3	PwithC	I1,I2,I3,I4	**Task13**	PwithC	I1,I2
Task5	DM	M1,M2,M3,M6	**Task14**	DM	M1,M2,M3
Task6	PwithC	I1,I2,I3	**Task15**	DM	M1,M2
Task7	DM	M1,M2,M3,M4,M5,M6	**Task16**	PwithC	I1,I3
Task8	DM	M2,M3,M5,M6	**Task17**	DM	M1,M2,M3,M4
Task9	PwithC	I1,I2,I3,I4	**Task18**	PwithC	I1,I3,I4
Task10	PwithC	I1,I2,I3,I4	**Task19**	DM	M1,M2,M3,M5,M6
Task11	PwithoutC	A1,A2,A4,A5			

Note. Please see Appendix 1 for the M1,M2,M3,M4,M5,M6,I1,I2,I3,I4,I5,A1,A2,A4,A5; DM: Doing Mathematics, PwithC: Procedures with Connections, PwithoutC: Procedures without connections

According to Table 3, Task11 was found to be the only one that fits under the low-level cognitive demand. Task11, known as the Ashure[1] task, involved solving percentages-related problems and linked the Ashure recipe and ingredients with the concept of percentages. Nevertheless, the directions necessitated limited cognitive demand since they clearly stated the operation to be employed, emphasized the production of the correct answer, and simply requested an explanation of the process utilized. Figure 1 illustrates the situation in Task11.

[1]Ashure, a renowned dessert in Türkiye and the Middle East, is conventionally made during the month of Muharram. The composition of this product includes a blend of various types of grains, legumes, and dried fruits. Ashure is considered a sign of cultural exchange and symbolizes peace and abundance.

Ingredients	Percentage
Wheat	73%
Sugar	18%
Legumes (Dry beans/Chickpeas)	7%
Dry fruit	1,5%
Cookie (Peanut/Hazelnut/Walnut)	0,5%

Students are divided into four groups. Each group fills in the blanks in the table by discussing among themselves.
Let's find the grams of the Ashure we will create with the help of the table above and fill in the blanks in the table. Let's place the results we found in the table. Then let's answer the questions below the table.

Groups	Amount of wheat	Amount of sugar	Amount of pulses	Amount of dry fruit	Amount of cookie	Amount of Ashure created
1.group			700 gr			
2.group						100gr
3.group					1050 gr	
4.group		900 gr				

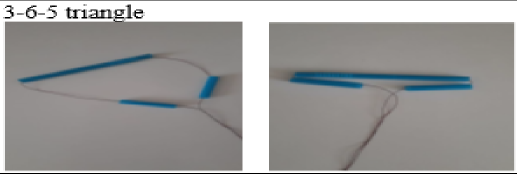
Figure 1. *The Low-Level Cognitive Demand Instructions of Task11*

Table 3 classifies mathematical tasks with high-level cognitive demand into two categories: 'doing mathematics' and 'procedures with connections,' with each category including an equal number of tasks. Task7, which falls under the 'doing mathematics' category, satisfies this level's characteristics. Task7 aims to improve students' comprehension of concepts about a right circular cone, along with their mathematical thinking and problem-solving skills, by utilizing dynamic geometry software and tangible materials (Appendix 2). This task requires students to employ non-algorithmic approaches to tackle complex problems as well as to formulate and implement their ways of solving geometric shapes and their nets. As students analyze the geometric characteristics of the cone, they explore mathematical connections and evaluate various measurements while constructing concrete models. This procedure requires tremendous cognitive effort and involves students constantly monitoring and modifying their cognitive processes. Task7 actively encourages students to evaluate and control their own learning and thinking processes, thereby expanding and deepening their mathematical thinking abilities.

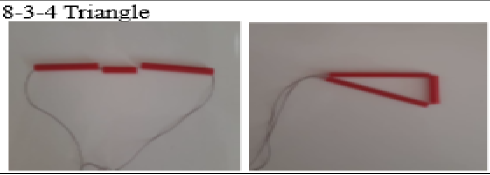
Task19 requires students to examine the possibility of constructing a triangle using three side lengths that are randomly chosen (Figure 2). In this process, students are provided the opportunity to utilize concrete models (constructing triangles with straws) and visual/dynamic representations (utilizing the GeoGebra application) to comprehend the triangle inequality theorem. This enables students to fully investigate and grasp mathematical concepts. Throughout the task, students have to evaluate their observations, analyze the outcomes, and arrange their learning strategies accordingly. In addition, they must conduct experiments using the given side lengths to determine if a triangle can be constructed and then make generalizations based on these experiences. This task demands students to critically examine the problem without relying on a pre-established formula or algorithm and devise their own strategies for solving it. This process requires a significant cognitive effort.

Students are asked to bring scissors, ruler and rope.
Students are divided into groups of four.
1. Cut the specified lengths from the straws
a-3-6-5
b-8-3-4
c-5-2-9
d-6-3-8
2. Write down the lengths in your notebook from smallest to largest.
3. Pass the string inside the straws
4.Has a triangle been formed? Didn't it occur? If it hasn't occurred, why do you think it hasn't occurred?

3-6-5 triangle



8-3-4 Triangle



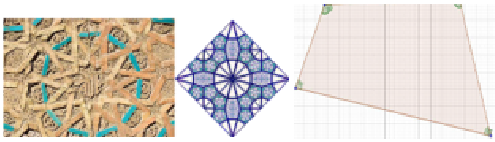
5. Repeat the first four steps with the given lengths and fill in the table below.

Order of edges	Sum of two selected side lengths	Difference of two selected side lengths	Third side length	Has a triangle been formed?
3-5-6	9	3	5	Formed
3-4-8	11	5	4	Not formed
2-5-9	7	3	9	Not formed
....	...	..	..	

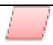
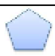
6. Is there a connection between the side length of triangles and whether a triangle can be formed?
7. According to the table, is a triangle formed when the length of the third side of the triangle is less than or equal to the difference in length of the two selected sides?

Figure 2. *Examples From Task19's Instructions*

Task 3, categorized under the 'doing mathematics' CDL, centers around the processes for identifying the diagonals and interior angles of polygons, as well as computing their sums (Figure 3). For this task, students must calculate and generalize the number of diagonals and total interior angles of polygons, presenting their findings with a formula while evaluating their thinking processes. This process requires the utilization and cultivation of problem-solving, analytical, and conceptual thinking abilities, surpassing a purely algorithmic approach. Thus, Task5 necessitates a substantial level of cognitive effort.



Find the sum of the interior angles of the polygon given above. Make a note in the appropriate place in the table.
Fill in the table appropriately with the data you obtained.

Name of the polygon	Shape	Number of sides	Sum of Internal Angles	Number of diagonals drawn from a corner of the polygon	Number of triangles formed in the diagonal drawn from a corner
Quadrilateral					
Pentagon					
Hexagon					
Heptagon …n-gon					

Find the sum of the interior angles of the polygon whose interior angles are not given below by drawing a diagonal from one corner.

1.What is the relationship between the number of sides and the number of triangles formed by diagonals drawn from a corner? Explain.
2. What is the relationship between the number of sides of polygons and the sum of interior angles? Explain.
3. Examine the table you filled out. Find the general expression that will give the sum of interior angles for the n-gene.
4. The triangle is not given in the table, what do you think is the reason? Do you think a triangle has a diagonal? If so, how many?

Figure 3. *Examples of Visuals and Instructions from Task5*

Task14 is designed as a ‘doing mathematics’ level task that requires students to identify lines of symmetry, analyze reflection patterns in motifs, and explore reflection relationships (Figure 4). Students explore the concepts of reflection and symmetry in Islamic art through motifs and patterns, while visually analyzing and deeply comprehending these relationships. This process provides students with a rich experience to comprehend, relate, and apply mathematical concepts. Furthermore, it necessitates students to actively monitor and regulate their own learning processes, thus promoting the growth of complex thinking and problem-solving skills.

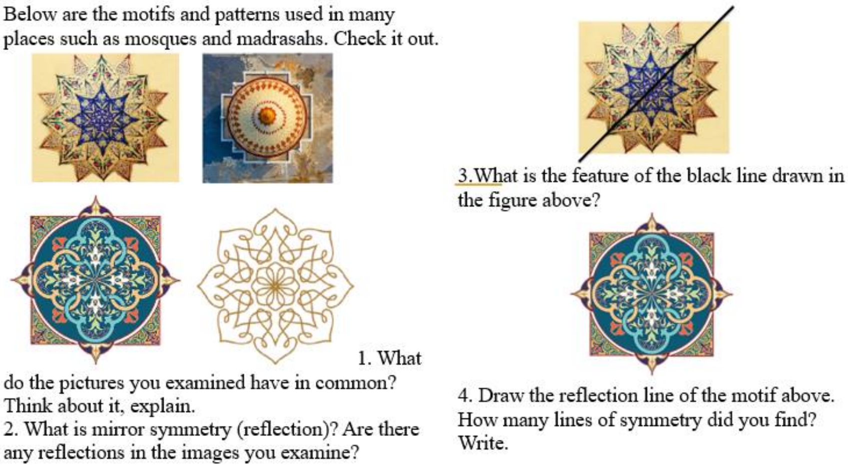


***Figure 4**. Examples from Task14’s Instructions*

Task15, categorized within the ‘doing mathematics’ CDL, offers a comprehensive and engaging learning opportunity that allows students to grasp mathematical relationships conceptually through hands-on models and analysis of their discoveries. This task provides students to experiment with constructing triangles using rods, exploring which side lengths can form a triangle and which cannot, with the aim of discovering the concept of the triangle inequality (Figure 5). This process requires students to explore, understand, and apply mathematical concepts, necessitating the use of complex thinking and problem-solving skills.

1. Students are divided into groups of 3-4 people.
2. Sticks of different lengths are distributed to each group (4,5,6,7,8,9,10,10,5,11,12 cm sticks)
3. A table is prepared to record the lengths of the sticks forming or forming a triangle. And the table is distributed to each group. (In the table below, fill in the required information in the columns by typing + if a triangle is formed, or - if it is not a triangle).

Rod lengths	triangle formed	Addition and subtraction results of other edges except the 1st selected edge	Addition and subtraction results of other edges except the 2nd selected edge	Addition and subtraction results of other edges except the 3rd selected edge	Did not form triangle	Addition and subtraction results of other edges except the 1st selected edge	Addition and subtraction results of the other edges except the 2rd selected edge.	Addition and subtraction results of the other edges except the 3rd selected edge
3 4 5	+	3 selected - differences: 1 Totals: 9	4 selected - Differences: 2 Additions 8	5 selected - Differences: 1 Totals: 7				
	.....	....	.....	.....				

4. Students are asked to create triangles using these sticks. Rod lengths are recorded.
5. Using the data recorded in two columns (rod lengths - triangle formed - not formed), students are asked to keep the length of each side of the shapes in the other three columns constant and record the results of addition and subtraction operations with the lengths of the other two sides in the table. Each group is asked what kind of relationship they reached in the results they obtained.
6. They are asked to reach a generalization by giving ABC triangle on the board as shown below, keeping the length of each side constant and adding and subtracting with the lengths of the other two sides. It is asked what is the relationship between these edges and operations in the triangle called?

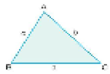


Figure 5. *Examples from Task15's Instructions*

The tasks at the 'procedures with connections' CDL involve following broad and general procedures instead of simple algorithms, making connections between multiple representations, applying mathematical thinking and skills, and requiring a significant amount of cognitive effort. Task3, Task9, and Task10 hold all of these characteristics. For instance, Task9 necessitates students to learn about the right circular cone, discern its basic elements, construct it, and draw its nets utilizing concrete materials (Figure 6). This task explores the cultural and practical aspects of mathematics in everyday life, using inspiration from conical hats originating from Asia. Students are instructed to construct a cone using the disk method, ensuring adherence to broad and general procedures associated with the fundamental conceptual ideas of the cone. The tangible models used facilitate students in acquiring a deeper understanding of mathematical concepts and ideas, as well as in establishing connections between multiple representations. Moreover, the process of applying mathematical thinking to a personal project using creativity requires cognitive effort. This process facilitates students' comprehension of how geometric shapes combine to create a cone and enables them to use this knowledge in practical situations, thereby actively engaging with the conceptual ideas that underlie the operations.

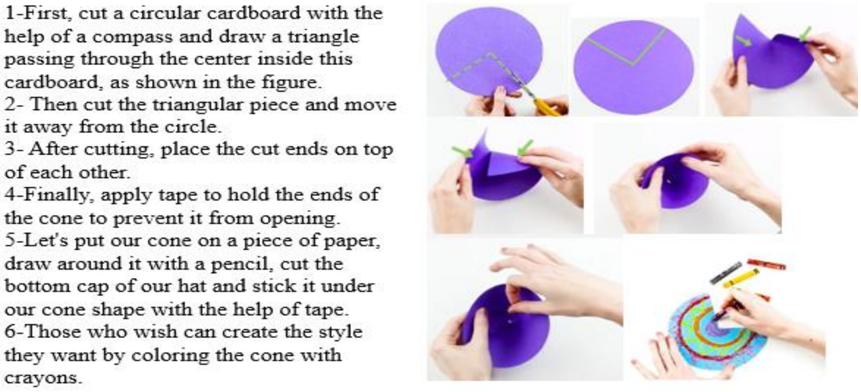


Figure 6. *Examples from Task9's Instructions*

Task6 is designed to instruct students on how to articulate the relative position of one point with another by employing direction and units of measurement, and by applying Piri Reis's World Map and real-life examples (Figure 7). This task necessitates students to understand the relative position of one point to another and apply this understanding in calculations involving direction and distance. The task enables students to utilize mathematical concepts in real-world situations. Additionally, it allows the use of mathematical thinking skills in everyday calculations of direction and distance. This demands students to possess a profound comprehension of mathematical concepts and to follow procedures associated with key conceptual ideas. Maps aid in the transition between visual and textual forms of knowledge, leading students to interpret the map and relate it with written explanations.

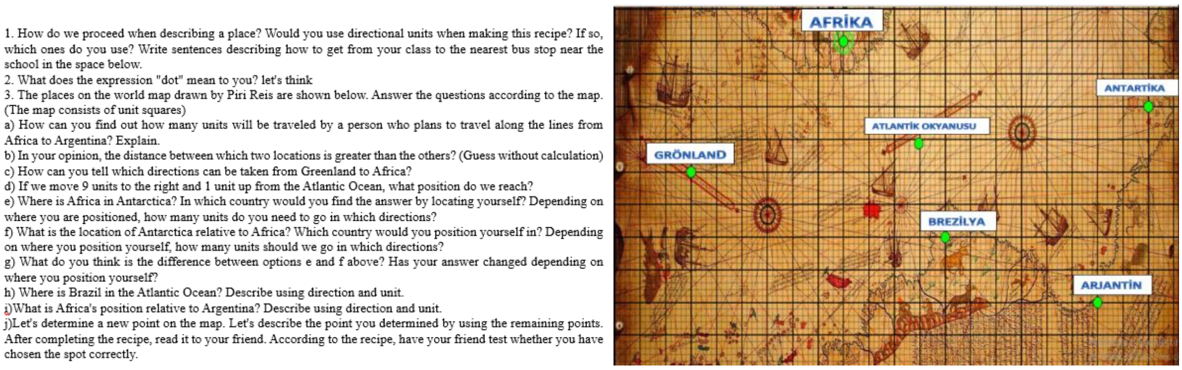


Figure 7. *Examples from Task6's Instructions*

Task16 guides students in comprehending and investigating the properties of sides and angles in a square using GeoGebra software (Figure 8). Through the use of GeoGebra, students can systematically construct the properties of the square's sides and angles, thereby strengthening their conceptual understanding of mathematical concepts. Additionally, the transitions between multiple representations are facilitated by the use of GeoGebra, as well as visualizations of the Iznik Green Mosque, baklava, and hand-knitted washcloths, resulting in a rich and interactive learning experience.

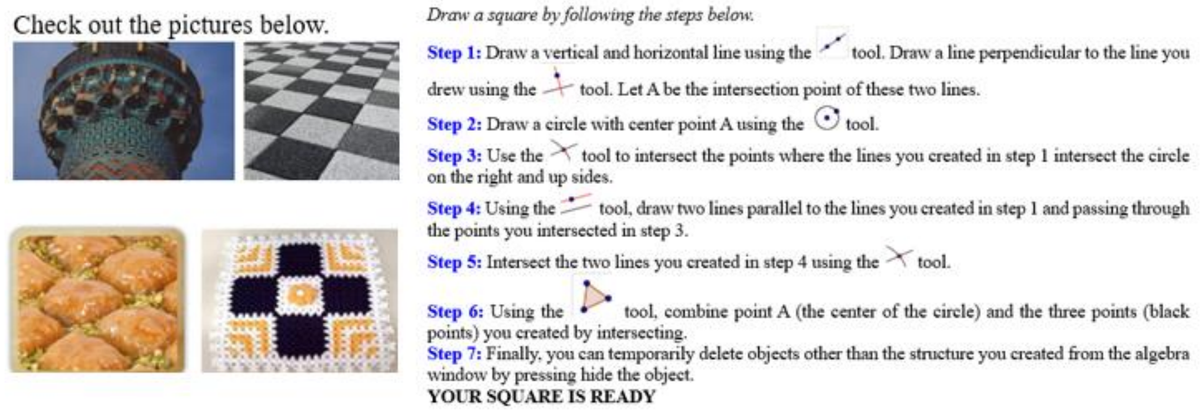


Figure 8. *Examples from Task16's Instructions*

4.2. PMTs' Skills in Integrating Culture and Mathematics in Designed Ethnomathematics Tasks

The ECD-MTR was employed to assess sixteen ethnomathematics tasks with high-level cognitive demands. In this framework, the eight criteria enumerated in Table 2 were used to classify each task in Table 4 as low, medium, or high. The total scores were then used to analyze the PMTs' abilities to integrate culture with mathematics. During the evaluation process, it was noted that half of the tasks examined were at the 'procedures with connections' CDLs, while the other half were at the 'doing mathematics' level.

Table 4. *PMTs Abilities to Integrate culture with Mathematics by Using ECD-MTR*

Criteria/Tasks		1	3	5	6	7	8	9	10	12	13	14	15	16	17	18	19
Integration culture and mathematics	**H**				x			x		x	x	x			x	x	
	M		x				x		x				x	x			x
	L	x		x													
	V					x											
Referring to a social situation	**H**										x						
	M							x									
	L																
	V	x	x	x	x	x	x		x	x		x	x	x	x	x	x
Providing daily life context	**H**				x		x	x			x				x		
	M		x											x			
	L	x		x		x			x	x		x	x			x	x
	V																
Addressing the relationship	**H**	x		x	x	x	x	x	x	x	x		x		x	x	x
	M		x									x		x			

between topics and concepts	**L**																
	V																
Using transition between representations	**H**	x		x	x	x	x	x	x	x	x	x	x		x	x	x
	M		x											x			
	L																
	V																
Being mathematically rich	**H**	x		x	x	x			x	x		x	x		x	x	x
	M		x				x				x			x			
	L							x									
	V																
Being cognitively demanding	**H**	x		x	x	x			x	x	x	x	x		x	x	x
	M		x				x	x									
	L													x			
	V																
Being embedding in cultural activity	**H**				x			x		x	x	x			x	x	
	M			x			x										
	L	x	x						x				x	x			x
	V					x											
Score		15	13	16	21	13	17	20	16	19	23	18	16	12	21	19	16
Decision		BB	BB	BB	BH	BB	BH	BH	BH	BH	BH	BH	BB	BB	BH	BH	BB

BH: Beyond Horizons, BB: Building Bridges, H: High, M: Moderate, L: Low, V: Vey Low

It is evident from Table 4 that ethnomathematics tasks provide a wide range of CDLs and cultural integration. The 'building bridges' category encompasses half of the ethnomathematics tasks with high-level cognitive demands (Task1, Task3, Task5, Task7, Task10, Task15, Task16, Task19), while the other half (Task6, Task8, Task9, Task12, Task13, Task14, Task17, Task18) are classified as 'beyond horizons' by using ECD-MTR. Within the CDL of 'doing mathematics', there are two tasks classified as 'beyond horizons' (Task8, Task17), and six tasks classified as 'building bridges' (Task1, Task5, Task7, Task14, Task15, Task19). In the procedures with connections category, five tasks are categorized as 'beyond horizons' (Task6, Task9, Task12, Task13, Task18), and three are 'building bridges' (Task3, Task10, Task16). Furthermore, ECD-MTRs' evaluation has been ascertained that the PMTs have not created any ethnomathematics tasks classified under the 'initial explorations' (0-7 points) category.

Task17, classified as having a CDL of 'doing mathematics', is categorized under the 'beyond horizons' category by using ECD-MTR in Table 4. The cultural architects of Task17 contextualize the concept of translation by referencing historical structures such as the Zeynel

Bey Tomb in Hasankeyf and the Iznik Mosque Minaret in Bursa, as well as culturally significant items such as knitted sweaters, vests, kilims, glass paintings, archery, and traditional children's games. The provided examples demonstrate the presence of translational movements in cultural objects, as depicted in Figure 9. Nevertheless, ECD-MTR shows that Task17 fails to tackle any particular social situations .

Figure 9. *Cultural Elements Included in Task17*

Task19, which is categorized by ECD-MTR under the 'building bridges' category and the 'doing mathematics' CDL, synthesizes mathematical concepts with cultural elements at the initial phase of the task. Bridge builders of Task19 link mathematical instruction with cultural and historical elements, including the Turkish triangle and the discoveries of Omar Khayyam (Figure 10). In the introduction of the task, the Turkish triangle, used during the Seljuk period, is defined, and questions about how Seljuk architects utilize this geometric shape are posed to stimulate students' thinking. Nevertheless, the cultural connection established at the beginning of the task does not persist in the subsequent sections, as the task transitions towards a more mathematically analytical and applicable approach. Students can acquire the concept of triangle inequality through the use of the GeoGebra application and tangible materials (straws) in Task19. In this manner, students are asked to find the side lengths of triangles, record them in a table, and are encouraged to explore the relationships between them. This enables smooth transitions between different representations and concepts, while it does not illustrate the practicality of the concept with particular instances from everyday life. Task15 is assessed at the 'doing mathematics' level and is classified under the 'building bridges' category as well. Nevertheless, the bridge builders of Task15 do not incorporate mathematics into a cultural context, as they have only provided a few instances of patterns and motifs from woven products (Figure 10).

Figure 10. *Cultural Elements Included in Task19 and Task15*

Task13, classified as a CDL of 'procedures with connections', falls under the 'beyond horizons' category of ECD-MTRs. This task entails the examination of a menu featuring ancient Turkish dishes and is one of the two most highly rated tasks devised by cultural architects that reflect social situations (Appendix 3). The cultural architects of Task13 sought for students to understand basic set concepts by categorizing the dishes presented on the menu. The process emphasizes the recognition of cultural heritage and the investigation of various representations of sets, including the null set, the cardinality of a set, unions, and intersections.

Practical applications, such as calculating the number of items in food categories or identifying common dishes across various categories, aid students in comprehending how mathematics can be integrated with daily life and culture. The task provides students with the opportunity to utilize diverse representations, including grouping, Venn diagrams, and listing techniques. This step is essential in the advancement of mathematics thinking skills. Task13 offers a learning experience that is intellectually stimulating and focuses on improving students' problem-solving, critical thinking, and creative thinking skills. This exemplifies the PMTs’ remarkable ability to create and plan at every stage of the task as cultural architects.

Task12, included within the CDL of ‘procedures with connections’ and ECD-MTRs’category of ‘beyond horizons’, is a study focused on comprehending geometric concepts employed in architecture and reflecting societal aesthetic values (Figure 11). The designers of Task12 aimed to investigate the relationships between special quadrilaterals based on their characteristics. As part of the task, students must create quadrilaterals using craft paper and then record the lengths of the sides and measurements of the angles in a table. This method has afforded students the opportunity to develop linkages between tangible models, algebraic representations, and tabular representations. Mathematics and culture have been connected through the use of diverse cultural components, including the Konya Karatay Madrasa, Ancient Egyptian Mathematics, the Ince Minare Medrese, the Sivas Çifte Minare Medrese, wall and dome motifs, and carpet and kilim patterns. The task also seeks to illustrate to students the application of geometry in the realm of aesthetics and design by examining the geometric patterns present in the entrance gates and architectural details of madrasas. Consequently, the task requires the establishment of a bridge between mathematics and art.

Figure 11. *Cultural Elements Included in Task12*

It has been determined that bridge builders, who devise tasks within the ‘building bridges’ category and the ‘procedures with connections’ CDL, frequently fail to establish connections with everyday life. Task10 aims to investigate the relationship between the side lengths of a triangle and the measurements of the angles opposite to those sides. Triangle shapes from Turkish weaving kilim motifs have been utilized in this process. These geometric shapes are presented as an indirect way of linking mathematical thinking with daily life (Figure 12). Furthermore, the inclusion of various triangles and visual representations in tables enables students to comprehend concepts from different viewpoints and perceive the connections between them. Students must engage in meticulous observation, analysis, and comparison to complete the task. Comprehending the reason why the side opposite the greater angle is longer necessitates a cognitive effort.

In Turkish culture, weaving has become an art under the influence of the nomadic lifestyle. For this reason, kilim motifs have survived to this day as an important cultural value. The motifs woven on carpets and rugs in Turkish weaving give information about the lifestyle of that period.

Figure 12. *Cultural Elements and Connections to Daily Life Included in Task10*

Task3 and Task16 have demonstrated the lowest performance in the CDL of ‘procedures with connections and the category of ‘building bridges’. These tasks exhibit inadequacies in addressing the connections between concepts and subjects, using real-life situations, and aiding the transition between different representations. Task3 aims to acquaint students with the right circular cone, ascertain its basic elements, construct it, and draw its nets. However, there has been no direct use of tables or graphs and attempts to connect these examples of right circular cones with culture and everyday life have been insufficient. Similarly, Task16 employed a cultural element, such as the Iznik Green Mosque, to introduce square shapes; however, this served as a conceptual entry point rather than a substantive integration into the cultural aspects of mathematics. Efforts were made to establish a connection with ordinary situations by incorporating sights like baklava and playground floors, however, these endeavors proved unsuccessful. The use of dynamic software such as GeoGebra allowed for the construction of squares, enabling the exploration of their geometric features. Nevertheless, there is a dearth of information regarding the use of tables and graphs or the transitions between representations.

5. RESULTS, DISCUSSION AND RECOMMENDATIONS

This study extensively examines the proficiency of PMTs in incorporating both culture and mathematics in the ethnomathematics tasks they design. To achieve this objective, groups of PMTs developed nineteen ethnomathematics tasks. The research was conducted in three hierarchical stages. The initial phase involved assessing the tasks’ adherence to the principles of activity design. The CDLs of the tasks that adhered to the activity design principles were determined in the second phase, employing the theoretical framework of Smith and Stein (1998). In the last stage, the integration of culture with mathematics in tasks with high CDLs was examined using the ECD-MTR, developed by the researchers drawing on the theoretical frameworks of Matthews et al. (2013) and NCSM (2020). The inclusion of ethnomathematics tasks in an elective course had a profound impact on the experiences of PMTs. Understanding the experiences and perspectives of PMTs as they transition from the role of student to teacher is crucial. Hence, this study offers significant perspectives on the existing discussions regarding mathematics teacher education and emphasizes the potential of ethnomathematics tasks to influence forthcoming educational practices.

The ethnomathematics tasks created by PMTs display differences in how well they follow activity design principles. The exclusion of some ethnomathematics tasks from evaluation provides critical insights into the PMTs' design processes, highlighting deficiencies in compliance with these principles. The necessity of enhancing the knowledge and skills of PMTs in activity design is clear from this situation. The significance of emphasizing the principles of activity design in teacher education programs and offering more comprehensive training to PMT for the development of well-designed activities is emphasized by this discovery. Research indicates that the development of activity design skills in teacher training programs in accordance with the principles of activity design, is essential for the creation of meaningful learning experiences (Barnett et al., 2024; Radmehr, 2023; Simon & Tzur, 2004). PMTs can improve their capacity to develop learning tasks that foster critical thinking and engagement among students by adhering to established principles and receiving comprehensive training. This supports the initial professional development of PMTs, as indicated by the NCTM (2000), by providing them with necessary tools to increase students' interest in mathematics.

The CDLs of the tasks devised by PMTs, which have been determined to adhere to the principles of activity design, are variable. It has been determined that only one of these tasks is at a low-level cognitive demand. This task demands minimal cognitive effort, typically adhering to specific algorithms or formulas to determine the correct answer and emphasizes performing fundamental operations rather than facilitating a profound comprehension of mathematical concepts. This result foresees that PMTs tend to provide students with superficial knowledge in their future practices (Agterberg et al., 2022; Estrella et al., 2020; Ni et al., 2018; Parrish & Bryd, 2022; Stein et al., 1996). Conversely, the remaining sixteen tasks are notable by their higher cognitive demands. By completing these tasks, students can deepen their comprehension of mathematical concepts and improve their problem-solving abilities, all while cultivating their analytical, conceptual, and complex thinking capabilities. Some studies parallel to this work suggest that PMTs are capable of designing tasks that necessitate high cognitive demands (Coşkun et al., 2023; Creager, Snider & Parrish, 2021; Estrella et al., 2020; Norton et al., 2012), while others, particularly those focused on distance education, indicate that tasks created by PMTs tend to be at a low-level cognitive demand (Bağdat, 2019; Gülbağcı-Dede, Özen-Ünal & Yılmaz, 2023; Whittington & Teknumru-Kısa, 2020). It is advised that tasks that necessitate higher-level thinking be chosen and implemented in classrooms to guarantee students' success (Norton & Kastberg, 2012; Redfield & Rousseau, 1991). Additionally, Levenson (2022) underscores the significance of considering the cognitive requirements of students in the learning environment and the potential of tasks to enhance mathematical creativity. The current study demonstrates that PMTs prioritize the development of students' complex problem-solving skills and in-depth comprehension of mathematical concepts by primarily designing tasks with a high cognitive demand.

The equal distribution of sixteen ethnomathematics tasks between the 'procedures with connections' and 'doing mathematics' CDLs reflects the ability of PMTs to provide students with a diverse range of mental challenges. The tasks at the 'doing mathematics' level illustrate the PMTs' emphasis on comprehensively enhancing mathematical abilities, supporting the development of mathematical thinking skills, and engaging students thoroughly with mathematical concepts and processes (Boston & Smith, 2011; Henningsen et al., 1997; Ruk, 2020; Stein et al., 1996; Stein et al., 2000). Tasks at the 'procedures with connections' level demonstrate that PMTs prioritize the development of learning environments that consistently encourage students to participate in critical thinking and deeply interact with mathematical concepts (Estrella et al., 2020; Henningsen et al., 1997; Ni et al., 2018; Ruk, 2020; Stein et al., 1996; Wakhata et al., 2023). This dual approach reflects PMTs' proactive attitudes in creating student-centered and participatory learning environments, enriching mathematics education, and potentially enhancing students' analytical thinking and applied knowledge integration skills. Stein et al. (2000) underline the importance of educators using tasks to improve students' mathematical ability as a crucial pedagogical strategy. Indeed, Stein et al. (1996) and Ruk (2020) acknowledge that it is difficult to maintain cognitive demand when performing tasks and that specific measures are necessary to sustain cognitive demand. Ruk (2020) asserts that when tasks are designed at the 'procedures with connections' level, it guarantees the sustained high cognitive demand during task execution. This promotes a profound involvement with mathematical concepts and prevents the common decrease in cognitive engagement that occurs during tasks. Therefore, this balanced distribution of designed tasks demonstrates that PMTs possess strong theoretical and practical pedagogical approaches, indicating their capacity to utilize innovative and efficient resources in their future teaching practices.

The ethnomathematics tasks, assessed using a rubric devised by researchers, have been categorized into three groups according to their degrees of cultural integration: 'initial explorations,' 'building bridges,' and 'beyond horizons'. In general, the results suggest that PMTs have been somewhat successful in incorporating cultural and historical elements into ethnomathematics tasks. However, the level and depth of this integration differ across different tasks. Research findings indicate that PMTs typically demonstrate moderate to high levels of cultural integration in ethnomathematics tasks but have not designed tasks at the 'initial explorations' level. It implies that they usually prioritize higher levels of integration. In tasks categorized under 'building bridges,' PMTs often limit the connection between culture and mathematics to examples, leading to superficial relationships. This situation may arise from a deficiency in understanding how to incorporate cultural contexts deeply into mathematics (Sunzuma & Maharaj, 2019; Supriyadi et al., 2023). Çenberci and Horzum (2023) conducted research that demonstrates PMTs' awareness of the interaction between mathematics and culture. However, they frequently concentrate on the mathematics within various cultures rather than the culture within mathematics. This finding could explain the insufficiencies of PMTs in deeply integrating cultural contexts into mathematics teaching. To improve this situation, it is recommended that, during the preservice period, greater emphasis be placed on the potential of ethnomathematics (Weldeana, 2016) and the significance of cultural relevance in mathematics instruction (Ergene et al., 2020) by providing more effective and comprehensive courses on the history of mathematics. This would afford PMTs the chance to get a profound comprehension of the historical and cultural backgrounds of mathematical concepts and incorporate this knowledge into their teaching. Workshop activities and practical training programs that offer PMTs opportunities to practice cultural integration could be beneficial. Moreover, It is noteworthy that the majority of bridge builders neglect social contexts, whereas only two cultural architects recognize and develop this connection. This suggests that cultural architects are connecting mathematical thinking with social consciousness due to their aspiration to connect mathematics with real-world situations and their endeavors for more impactful instruction. In this case, the experiences of bridge builders could be seen as a missed opportunity for fully integrating cultural and historical elements in the design of ethnomathematics tasks.

Tasks at the 'doing mathematics' CDL have reached the 'beyond horizons' category, demonstrating the PMTs' proficiency in higher-level mathematical thinking and cultural integration. This indicates that cultural architects possess the potential to teach mathematical thinking comprehensively. Conversely, particular tasks, especially those at the 'doing mathematics' level, utilize cultural contexts only as a starting point and focus on mathematical analysis in later stages. This situation highlights the significance of maintaining cultural integration throughout the task (Orey & Rosa, 2008). Most tasks at the 'doing mathematics' level are classified as 'building bridges', indicating that complete cultural integration has not been attained. This suggests that the nature of 'doing mathematics' tasks necessarily necessitates involving students in more complex mathematical thoughts, which may result in less extensive exploration of cultural connections. Nevertheless, Akintunde et al. (2019) emphasized that when emphasizing complicated mathematical ideas, it is important not to overlook cultural aspects. Conversely, most of the tasks that at the 'procedures with connections' CDL are classified as 'beyond horizons'. These tasks address cultural integration more intensely and show that PMTs are capable of more integratively blending cultural contexts with mathematical content. This situation exemplifies that including cultural components in mathematics instruction is not only instructive but also a potent strategy that actively involves students and deepens their comprehension of mathematical concepts (Barton, 1996; Shultz et al., 2023; Sunzuma & Maharaj, 2019; Supriyadi et al., 2023). Therefore, although PMTs generally demonstrate a positive picture to incorporate history and culture in ethnomathematics tasks, there is a need for additional improvement in the extent and depth of this integration. In this direction, PMT training programs should provide more opportunities for integrating cultural contexts in high cognitive demand tasks and for relating mathematical concepts to everyday life and social situations, encouraging the development of these skills (Jia & Nasri, 2019; Kondo, 2022). This will allow students to improve both their cultural awareness and mathematical thinking skills. Therefore, the design of ethnomathematics tasks can be regarded as an effective tool that allows PMTs to develop both their mathematical and cultural pedagogical skills (Sunzuma & Maharaj, 2019). Improving the skills of PMTs to effectively employ cultural elements in mathematics instruction can greatly enhance their future teaching practices. Subsequent research should investigate the effects of these tasks on classroom settings and undertake more studies to enhance the CDLs.

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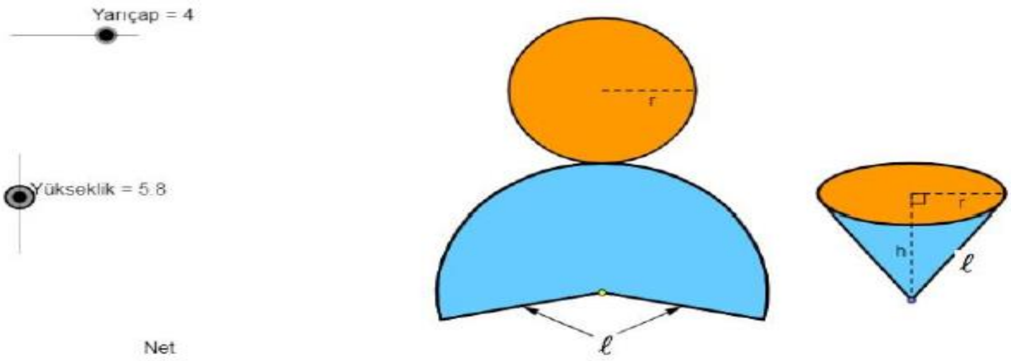
Appendix / Apendices

Appendix 1. Framework of CDLs

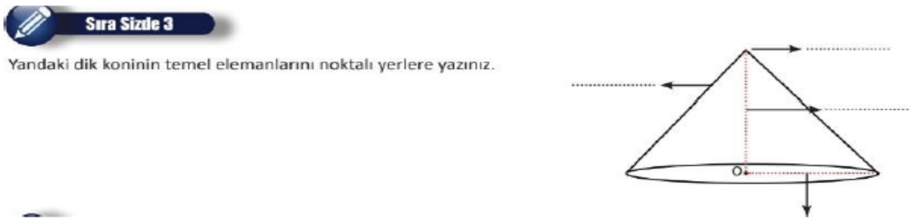
Level of cognitive demands		Characteristics
Low-Level Cognitive Demands	*Memorization*	**E1.** Involve either reproducing previously learned facts, rules, formulas, or definitions *or* committing facts, rules, formulas or definitions to memory. **E2.** Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. **E3.** Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be re-produced is clearly and directly stated. **E4.** Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced.
	Procedures without connections	**A1.** Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task. **A2.** Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it. **A3.** Have no connection to the concepts or meaning that underlies the procedure being used. **A4.** Are focused on producing correct answers instead of on developing mathematical understanding **A5.** Require no explanations or explanations that focus solely on describing the procedure that was used.
High-Level Cognitive Demands	*Procedures with connections*	**I1.** Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. **I2.** Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. **I3.** Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. **I4.** Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develops understanding.
	Doing Mathematics	**M1.** Require complex and non-algorithmic thinking - a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example. **M2.** Require students to explore and understand the nature of mathematical concepts, processes, or relationships. **M3.** Demand self-monitoring or self-regulation of one's own cognitive processes. **M4.** Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. **M5.** Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. **M6.** Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Appendix 2. English version of Task 7

Activity Sheet



(1st question) What are the basic elements of a right cone? Show it on the figure. Draw the expansion of the cone given in the figure.



(2nd question) Move the radius slider in the Geogebra activity. Explain how the cone changes when you move the radius slider.

(3rd question) Move the height slider in the GeoGebra activity. Explain how the cone changes when you move the height slider.

WHERE IS MY HAT?

How about designing a hat with us? Let's read the instructions and design a hat suitable for a basketball, football and tennis ball.

TOOLS AND EQUIPMENT TO BE USED

activity sheets
Scissors
tape measure,
Pen
Ruler
protractor
Glue
Ornamental materials (stamps, gilding, etc.)

Guidelines

STEP 1: Have students in each group measure the circumference of the basketball, football and tennis ball with the tape measure in their hands and record the measurement results on their activity sheets. (On your activity sheet, you should write 3 measurement results, 1 for the football ball, 1 for the basketball ball and 1 for the tennis ball.)

	MEASUREMENT RESULT
Basketball Ball	
Football Ball	
Tennis Ball	

STEP 1: Determine the radius of the hat you will design by establishing a relationship between the arc length of the circle segment that forms the lateral surface of the cone and the base circumference. (There was a formula where we took pi as 3, so now is the time to use it :))

STEP 2: What is the relationship between the main line of the cone and the radius of the circle segment? Explain. Find the length of the main line of the cone. Have each group fill in the line where their group is written.

Groups	Vertex angle	Radius	Main Line Length
1. Group	60 degrees		
2. Group	72 degrees		
3. Group	90 degrees		
4. Group	120 degrees		
5. Group	180 degrees		

STEP 3: Draw the expansion of the cone, for which you found the main correct length, on colored activity papers with the help of a ruler, following the video, and carefully cut out the shape you drew.(https://youtube.com/shorts/Ly0HHeEZV4s?feature=share)

STEP 4: Paste the shape you cut with the help of glue and obtain a cone.
STEP 5: Decorate the cone you obtained.

Note: Do these steps for a basketball, a football and a tennis ball to get 3 hats)

After all groups have finished designing hats, have the groups show the designed hats to each other and have one of the students in the group explain what the groups did during the process and answer the questions below.

Question 1) What is different about each group's party hat?

(Question 2) What is the reason why the heights of the designed hats are different from each other? Explain.

After all groups complete the activity, the activity sheets will be collected and the designed hats will be tried on the models.
The group that designs 2 hats according to the models will be the winner of the event.

Appendix 3: English version of Task13

Activity

The teacher first creates a discussion environment in the class by asking the class "Do you know the old Turkish dishes? If so, can you briefly tell us how they are made?" Thus, students' attention is drawn to the lesson. Then, the activity to teach the subject begins. Master Faruk has prepared a menu based on the dishes eaten by the ancient Turks, but there are many complaints from customers about the menu he prepared.
Customers tell Master Faruk that the menu is very complicated and that they have difficulty finding the dishes they want. Below is the menu prepared by Faruk Usta. Let us help Faruk Usta in organizing the menu.

1st page • ***Talkan:*** *Made from barley and wheat flour, similar to tahini-molasses. It is sweet.* • ***Tutmaç Çorbası:*** *main ingredients are wheat flour and meat, oguz soup. It is a delicious and nutritious soup that was very popular during the period.* • ***Tabak Börek:*** *The mixture of minced meat, onion and salt is placed on the dough rolled out in a square shape and folded into a bundle and pressed. Then, the dough is boiled, drained and served with heated butter and yoghurt.*	*2nd Page* • ***Oğuz Börek:*** *The fried minced meat, onion and salt mixture is placed on the dough rolled out in a round shape and closed in a crescent shape. Then, the dough is boiled, drained and served with fried butter and yoghurt on top.* • ***Yarma Çorbası:*** *It is a soup prepared by seasoning cracked barley and wheat with yoghurt.* • ***Türmek:*** *It is a dessert made from flour, honey and almonds.*
3rd page • ***Kagut:*** *Boiled millet is dried and pounded and then used as flour. Thin it and add oil and sugar. It is a sweet dish.* • ***Tarhana Çorbası:*** *It is a soup prepared with tarhana, which is prepared in summer and served for consumption in winter.* • ***Uwa:*** *It is a dessert prepared by putting rice in cold water after cooking, draining it and adding sugar. It is consumed cold.*	*4th page* • ***Buxsi:*** *Almond grains are thrown into the cooked wheat, a slurry made with honey and milk is poured over it, and the wheat of the sour dish is eaten. The water is drunk separately. It is a sweet dish.* • ***Irmşık Börek:*** *It is a dish that uses flour, water, eggs and salt for the dough, adding sour cream to the dough, and boils it in a rectangular shape. It is served with* • *butter and, optionally, yoghurt with or without garlic on top.*

Question 1: Why might customers have difficulty reading the menu? What can be done to help the customer understand the menu and make a more comfortable choice? Discuss with your deskmate. Let's write down your thoughts in the empty space below and share them with our classmates.

Question 2: Let's redesign the menu below with our deskmate so that customers can find the dishes they want more easily.

1st page	*2nd page*
3rd page	*4th page*

Question 3: Is there a mathematical equivalent for the groups you created for the menu you designed? Let's discuss it with your deskmate. Share the ideas from the discussion with your class.

Question 4: What do concepts such as set, element, number of elements mean to you? After discussing it with your deskmate, let's share your thoughts with the class.

Question 5: Based on all the thoughts, let's define the following concepts in the most complete way.

Set:

Element:

Element number:

Question 6: Can a "drinks" cluster be created in the menu? If it could be created, what would be the number of elements? Let's discuss it with your deskmate. Let's share the ideas at the end of the discussion with the class.

Question 7: How can we represent sets mathematically? Let's share our ideas with the class.