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Investigation of Elementary School Mathematics Teachers' Problem-Solving Skills without Variables and Problem-Solving Strategies

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Abstract: Problems can be considered as complex structures involving concepts such as equations and variables. Different strategies are effective ways of problem-solving. Using concrete objects or logical reasoning may be sufficient to solve a problem. Examining strategies allows for a more in-depth analysis of teachers' problem-solving approaches and an understanding of how they act in different circumstances. In this context, the present study investigated the ability of elementary school mathematics teachers to solve mathematical problems without relying on variables. The study's primary purpose was to examine which strategies teachers use in solving problems without variables. The participants were 33 elementary school mathematics teachers with different professional experiences from 14 schools. A form of five open-ended word problems was used as a data collection tool, and a content analysis technique was used to analyze the data. The findings revealed that teachers had difficulty in solving the problems without variables. In addition, it was observed that those who did not use variables solved the problem with different strategies. They frequently preferred to guess and check, using logical reasoning, and make tables and lists strategies. In addition, others solve the problem by representing the variable with various symbols and labels. It is recommended that comprehensive research be conducted on teachers' problem-solving strategies, variable use, and the effects of these skills on student achievement.

Keywords: *Problem-Solving Strategies, Variables, Elementary School Mathematics Teachers*

1. INTRODUCTION

Problems are situations that people frequently encounter in their daily lives. It is generally defined as ambiguous situations that require critical thinking to solve. In mathematics, they are considered questions that need to be solved using theorems or rules. Problems attract attention because of their complexity and lack of a direct solution, unlike simple solutions that achieve a known result (Krulik & Rudnick, 1988; Posamentier & Krulik, 2020). They are based on higher-level cognitive processing, which requires organizing and classifying data and understanding the relationship between variables. Polya (2017) defines problem-solving as eliminating a problem by developing and applying appropriate strategies. Similarly, Altun (2014) states that problem-solving is the process of overcoming difficulties by using an individual's existing knowledge from a critical perspective. The main goal is to ensure that

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individuals have the capacity to develop effective strategies for the problems they encounter, which is one of the main goals of the mathematics curriculum (Baykul, 2014).

In 2012, the compulsory education period was extended from eight to twelve years, and the 4+4+4 system in education began to be implemented in Türkiye. Thus, elementary school mathematics teachers instead of classroom teachers became responsible for teaching middle school mathematics courses starting from the fifth grade (Gökkurt Özdemir, Koçak & Soylu, 2018b; MoNE, 2012). Moreover, Posamentier and Krulik (2020) indicated that there might be differences in problem solving problem-solving processes according to students' cognitive development levels. In addition to having problem-solving skills, teachers need to modify their problem-solving processes according to different levels. Piaget's theory of cognitive development can explain it. Piaget's theory, which forms the study's conceptual framework, explains how children deal with concrete and abstract concepts. According to Piaget's theory of cognitive development, students who pass from primary school to elementary school are still in the concrete operations period (7-11 years old) and cannot reach the abstract operations period. Even 5th-grade students are in the concrete operations period. 6th grade students, meanwhile, are in the transition phase from the concrete operations period to the abstract operations period and have not fully acquired the characteristics of the period (Daşcı & Yaman, 2014). In light of Piaget's theory of cognitive development, it is understood that students have not yet reached the level of grasping abstract concepts. This theory reveals that students in the concrete operational period have difficulty understanding abstract mathematical concepts (e.g., variables) due to their underdeveloped abstract thinking skills. Problems need to be solved with strategies appropriate to the cognitive development levels of students who are primarily in the concrete operations period (Çalışkan Dedeoğlu & Eğerci, 2021).

Abstract concepts are generally used in finding the unknown, expressing the relationships between data or the solution path followed to reach the answer in the problems prepared for the different learning outcomes in the curriculum (Gökkurt Özdemir et al., 2018a). However, students in the concrete operation period utilize various models while solving problems. It can be seen as an alternative way instead of using variables and symbols frequently used in solving problems because it is not cognitively possible for students in the concrete operations period to understand abstract concepts such as variables. In addition, among the results of many studies, students who use figures and diagrams in the verbal problem-solving process are more successful (Beitzel & Staley, 2015; Chu, Rittle-Johnson & Fyfe, 2017; Xing, Corter & Zahner, 2016). Also, organizing and expressing problem sentences descriptively allows students to perceive problems closer to real-life situations, i.e., more realistically and concretely (Hoogland et al., 2016).

Teachers are expected to solve problems without abstract concepts such as variables, especially at grade levels with students in the concrete operations period. This requires that the problems should be expressed in a way that is close to daily life, have different ways of solution, and include concretizing elements such as figures, diagrams, charts, pictures, and tables in the solutions. Concretizing elements provide an advantage to teachers in the problem-solving process in classifying the data in the problem or explaining the relationship between them and visualizing the objects (Neria & Amit, 2004; Yavuz & Yüca, 2017).

In the literature, it is seen that students have difficulty in understanding problems involving an abstract concept such as variables and grasping the meaning of variables (Akgün, 2009). They also have difficulties understanding the situation in the problem sentence and transferring it to the mathematical model (Gökkurt Özdemir et al., 2018a). These difficulties can be associated with the students' cognitive development levels, and Piaget's theory of cognitive development provides a useful framework for understanding this situation (Vosniadou, 2014). Jupri and Drijvers (2016) found that seventh-grade students aged 12 and 13, who were in the abstract operations period, had difficulty with verbal problems and stated that students had difficulty translating the problem into a symbolic language. This finding is related to the difficulties in problem-solving, which is the focus of the current study. Students in the abstract operations period -who take courses related to algebra- prefer to solve problems in alternative ways (estimation and control, strategy development) while solving problems (Ross et al., 2011). Campos and Costa (2019) reported that pre-service teachers had difficulty in using algebraic concepts in the relevant context. These findings show that pre-service teachers also have difficulties with abstract mathematical concepts and encounter similar cognitive barriers to students. Kaput (2017) emphasized the importance of mathematics teachers being aware of students' algebraic thinking abilities in the middle school period, which is the transition from the concrete operations period to the abstract operations period, considering Piaget's cognitive development level. Therefore, it has been revealed that teachers need to organize problem-solving strategies at different levels, taking into account students' cognitive development levels. These results show that both students have difficulty in solving problems, and pre-service teachers have difficulty in expressing algebraic concepts. It is clear that teachers' problem-solving skills, especially processes related to abstract concepts, require further study. Problem-solving skills of pre-service teachers and teachers have been the focus of many studies. These include both problem-solving studies with different strategies (Çelebioğlu, Yazgan & Ezentaş, 2010; Usta, 2020) and non-variable problem-solving studies (Chazan, Sela, & Herbst, 2012; Gökkurt et al., 2016; Gökkurt Özdemir et al., 2018a; Öçal et al., 2020). Studies also investigate solving verbal problems without variables and with different strategies (Gökkurt Özdemir, Koçak & Soylu, 2018b).

As the importance of problem-solving in mathematics education has increased, there has been a need to analyze teachers' skills and knowledge. In curricula and reforms in mathematics education, it is frequently emphasized that problem-solving must be integrated into mathematics courses at all levels. Therefore, it becomes important to investigate teachers' problem-solving methods and competencies. Students may need to be able to solve the problem without using variables in accordance with their mental development level. However, most teachers may have difficulty problem solving due to their habit of using variables. Studies examining the problem-solving skills of primary and elementary school teachers without using variables are limited, although they are available in the literature. This study examines elementary school mathematics teachers' ability to solve problems without using variables and the strategies they use while solving problems. In addition, the research questions related to the purpose of the study are as follows:

1) How skilled are elementary school mathematics teachers at solving problems without using variables?

2) What alternative strategies do they use when variables are not available?

The present study has a unique value as it investigates the ability of elementary school mathematics teachers to solve problems both without variables and with different strategies. In addition, examining teachers' problem-solving skills without using variables will add more depth to studies in this area. Developing students' problem-solving skills with strategies appropriate to their cognitive development levels can increase success in mathematics classes and other academic areas. Also, it aims to contribute to taking action for the development of these skills. The present study is expected to help mathematics teachers understand the difficulties they may encounter in the problem-solving process, especially with students who are not at the abstract operations stage, and to contribute to the efficiency of the teaching process.

2. METHOD

2.1. Research Design

In the present study, the case study method, one of the qualitative research approaches, was used. A case study is a research method in which the researcher does not intervene in the examined situation, focuses on how and why questions and examines the event or phenomenon in its natural conditions. In addition, a specific subject or group is handled, and a general description of the situation is made (McMillan & Schumacher, 2010). The case study design was used because it provides the opportunity to examine the skills of primary school mathematics teachers in solving variable-free word problems and the strategies they use. Unlike other qualitative methods, a case study provides the opportunity to analyze teachers' individual experiences, strategies, and difficulties in problem-solving (Yıldırım & Şimşek, 2016). This design was preferred as the most suitable method for the purpose of the research, as it aims to observe teachers' problem-solving skills in a specific context (primary school mathematics teaching) and understand how they cope with various strategies.

2.2. Participants

The study was conducted with a total of 33 teachers, 12 males and 21 females, who were working as elementary school mathematics teachers in 14 different elementary schools. Maximum variation sampling method was used to select participants to ensure that the demographic characteristics of the teachers in the study (e.g. professional experience, education level and gender) were represented across a wide spectrum. Thus, teachers' problem-solving skills were examined from various perspectives. The professional experience of the teachers ranged between 1 and 15 years. Three of the teachers have master's degrees and the others have undergraduate degrees. In addition, four teachers were interviewed to enrich the data obtained. Interviews were conducted with one master's graduate and three undergraduates. Participants' identities were kept confidential, and consent forms were signed on a voluntary basis. Participants were clearly informed that the data obtained would be used only for research purposes, that their personal information would not be disclosed in any way, and that they would remain anonymous in reporting. The demographic variables of the participants are presented in Table 1. For confidentiality reasons, the symbol R was used instead of the researcher and $P_1, P_2, \ldots, P_{33}$ codes were used instead of the participants' real names.

Table 1. *Distribution of Participants according to Demographic Variables*

Variables	Categories	n	%
Gender	Female	21	63.6
	Male	12	36.3
Age	24-26	13	39.4
	27-29	11	33.3
	30-32	6	18.2
	32+	3	9.1
Education status	Undergraduate degree	30	90.9
	Master's degree	3	9.1
Professional working year	1-4	17	51.6
	5-8	12	36.3
	9-12	3	9.1
	12+	1	3
	Total	33	100.0

2.3. Data Collection Tool

A form consisting of 5 problems that can be solved without variables was used. In the preparation of the form, both the gains of the elementary school mathematics curriculum were taken as reference and a literature review was conducted. In addition, open-ended problems that can be solved with different strategies were included. In this context, firstly, a draft of 16 problems that can be solved without using variables was created. Content validity was checked by matching these problems with the learning outcomes in the curriculum. To ensure the validity of the form, the expert comments were obtained from 2 lecturers and 1 doctoral student who are experts in mathematics. As a result of the expert feedback, 7 problems were removed from the draft due to limiting the total number of problems and low scores. Four of the remaining problems were removed from the form because they measured similar objectives, and the final version consisted of 5 problems. A pilot study was conducted with 2 elementary school mathematics teachers, and feedback was received about the comprehensibility of the problems and the duration. The pilot study provided feedback on teachers' responses to the problems and the time it took to solve the problems, as well as providing data on the difficulty level of the problems. Based on the feedback, no corrections were required in the form and the form was applied as is. The final form, consisting of 5 problems, was applied to 33 teachers. Interviews were conducted with the participants whose solutions differed from the other participants. Teachers with different educational status and professional experience were included in the interviews. In these interviews, teachers were asked to explain their written answers to the problems in detail, to explain the parts that were difficult to understand and to justify the strategies they used. In this way, it was tried to determine how many different strategies were used to solve the problems. The interviews lasted approximately 20-25 minutes and were recorded with a voice recorder to analyze the data.

2.4. Data Analysis

Data analysis was carried out process gradually using content analysis techniques and descriptive analysis methods. Content analysis is a technique based on summarizing data by coding it. Its purpose is to explain the concepts and relationships in the data. Descriptive analysis summarizes and interprets the data according to themes supported by direct quotes. In the creation of the codes, (Posamentier & Krulik, 2020)'s problem-solving strategies and whether teachers use variables in problem solutions were taken into account. Draft codes were obtained from the teachers' written answers, and draft categories were obtained from the codes. Experts examined the draft codes and categories in the field of mathematics teaching to determine their suitability for the data. Irrelevant/unnecessary codes were eliminated, and new codes were added in line with the purpose of the study. The coding scheme was formed according to whether to use variables in the solution to solve the problem. To ensure the reliability of the coding, both researchers coded the data obtained independently. The percentage of agreement between the two raters was calculated by using the percentage of agreement of Miles and Huberman (1994) (agreement/agreement + disagreement), which was found to be 94%. The researchers agreed on the remaining 6% difference. In cases of disagreement, the relevant codes were reviewed as a result of discussions between the two researchers and the final decision was made by reaching a consensus. Some of the participants solved the problems without variables, while some of them solved the problems using variables. Therefore, categories specific to both types of solutions were determined. The codes and categories are presented in Table 2.

Table 2. *Categories and Codes*

Categories	Codes
Solving problem correctly (without using a variable)	Guess and check Using unit fractions or ratio Using models (area model, length model etc.) Using a logical reasoning strategy Using least common multiple Make tables and lists Working backward
Solving problem correctly (using a variable)	Using variables Representing the variable with symbols (e.g. □, Δ) Representing the variable with a name
Incorrect solving (with or without using variables) or no solution	Solving incorrectly (without using a variable) Solving incorrectly (using a variable) No solution

3. FINDINGS

This section presents percentage and frequency tables of the codes related to the answers given by elementary school mathematics teachers to the problems. For each problem, only related codes were included. In addition, the study's findings were elaborated on by direct quotations from the interviews with four teachers and their written answers.

3.1. Findings regarding the First Problem

Table 3 presents the distribution of the participants' answers to the first problem according to categories and codes.

Table 3. *Percentage and Frequency of the Categories and Codes related to the First Problem*

Categories	Codes	f
Solving problems correctly (without using a variable)	Guess and check	8
	Using models (area model, length model, etc.)	1
	Using a logical reasoning strategy	7
	Make tables and lists	2
Solving problem correctly (using a variable)	Using variables	3
	Representing the variable with symbols (e.g., □, ∆)	10
	Representing the variable with a name	2

More than half of the participants (54.5%) were able to solve the first problem without using variables. In addition, they mostly used guess and check and logical reasoning strategies. The answers of the participants who solved the first problem without using variables are presented in Figure 1.

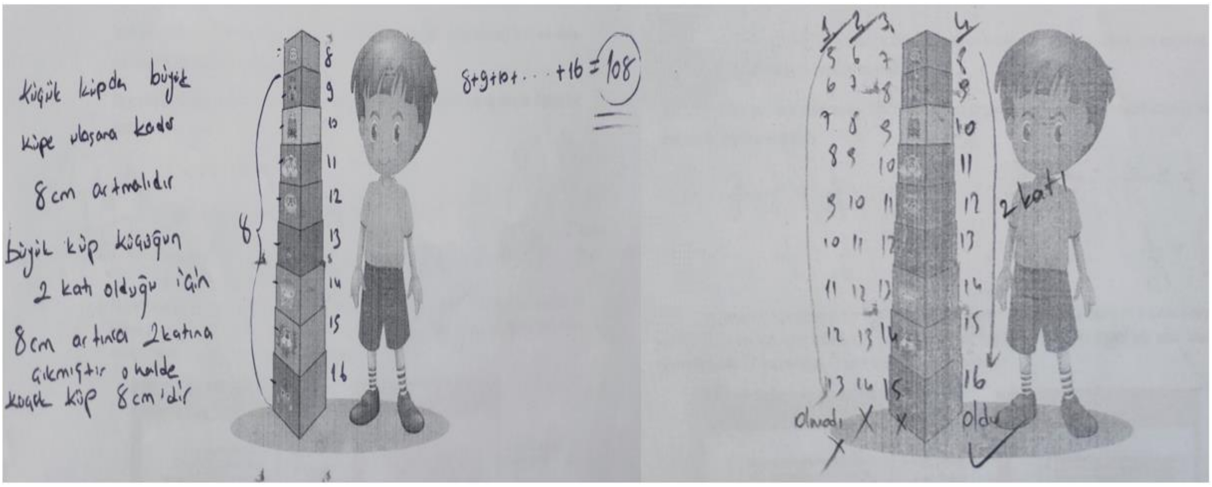

Figure 1. *P_1 and P_2 Answers to the First Problem*

P_1 suggested that the side length of the smallest cube must be 8 cm for 2 times the side length of the smallest cube to be equal to 8 cm, which is the length difference between the cubes. In the solution of P_2, the side length of the smallest cube was given numerical values starting from 5. It was seen that the ratio of the side lengths between the cubes approached 2 by estimating that values greater than 5 could be possible. It was checked until the value of 8 that the largest cube's side length was twice the smallest cube's. Both teachers solved the first problem correctly by choosing different strategies without using variables.

In addition, it was observed that all the remaining participants (45.5%) solved the problem correctly by using variables. Most of the participants who used variables solved the problem by using symbols such as □, ∆, etc. instead of variables. Other participants solved the problem by using a variable or representing the variable with a name. The solution of the participant who solved the first problem by representing the variable with various geometric symbols is presented in Figure 2.

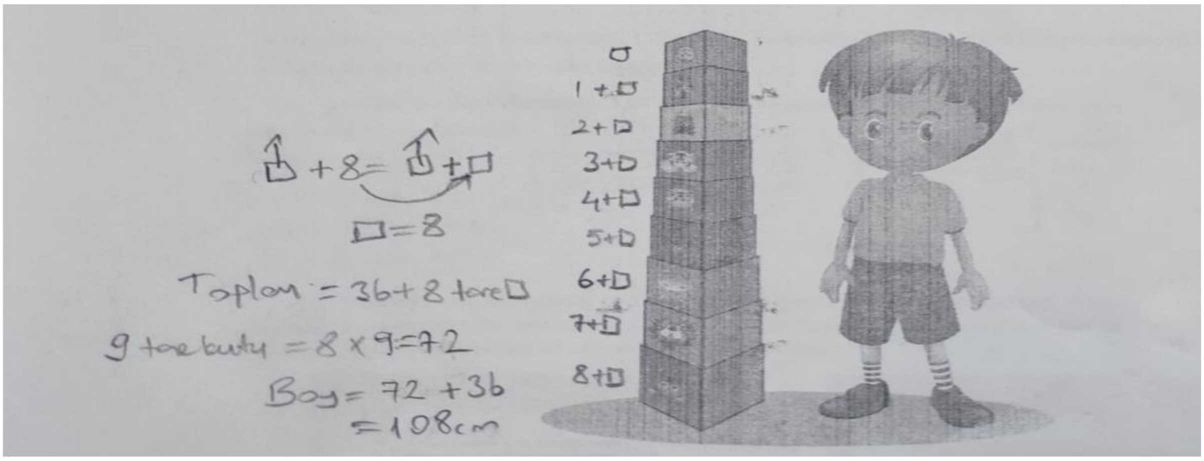

Figure 2. *P_6 Answers to the First Problem*

P_6 determined the side length of the smallest cube as a variable and established an equation. In the equation, the symbol □ was used to represent x. P_6 thought that the symbol □ used instead of x was not a variable and found the numerical value of the variable. It could be said that he made a mistake by thinking that he solved the equation (Total=36+8.□) without variables. The explanation of the P_6 support this.

R: I saw that you used symbols in your solution. What was the reason for using symbols?

P_6: I was asked to solve the problem without variables. Therefore, I used a box sign instead of x. Next, I determined the length of the small cube as □. By adding one centimeter downwards, I found the length of the largest cube as □+8. I equaled the length of the large cube given in the question to 2 times the length of the small cube as □+8=□+□. I found 'x' as 8. I can find the answer by adding 9 boxes with numbers from 1 to 8. Since I found one box as 8, 9□=72. Thus, I reached the answer 108 by adding 72 and 36.

3.2. Findings regarding the Second Problem

Table 4 presents the distribution of the participants' answers to the second problem according to categories and codes.

Table 4. *Percentage and Frequency of the Categories and Codes related to the Second Problem*

Categories	Codes	f
Solving problem correctly (without using a variable)	Guess and check	1
	Using logical reasoning strategy	11
	Make tables and lists	1
	Working backward	12
Solving problem correctly (using a variable)	Using variables	1
	Representing the variable with symbols (e.g. □, Δ)	3
	Representing the variable with a name	1
Incorrect solving (with or without using variables) or no solution	Solving incorrectly (using a variable)	1
	No solution	2

Three-quarters of the participants (75.7%) were able to solve the second problem without using variables. Moreover, they mostly used to work backward and logical reasoning strategies. The answers of the participants who solved the second problem without using variables are presented in Figure 3.

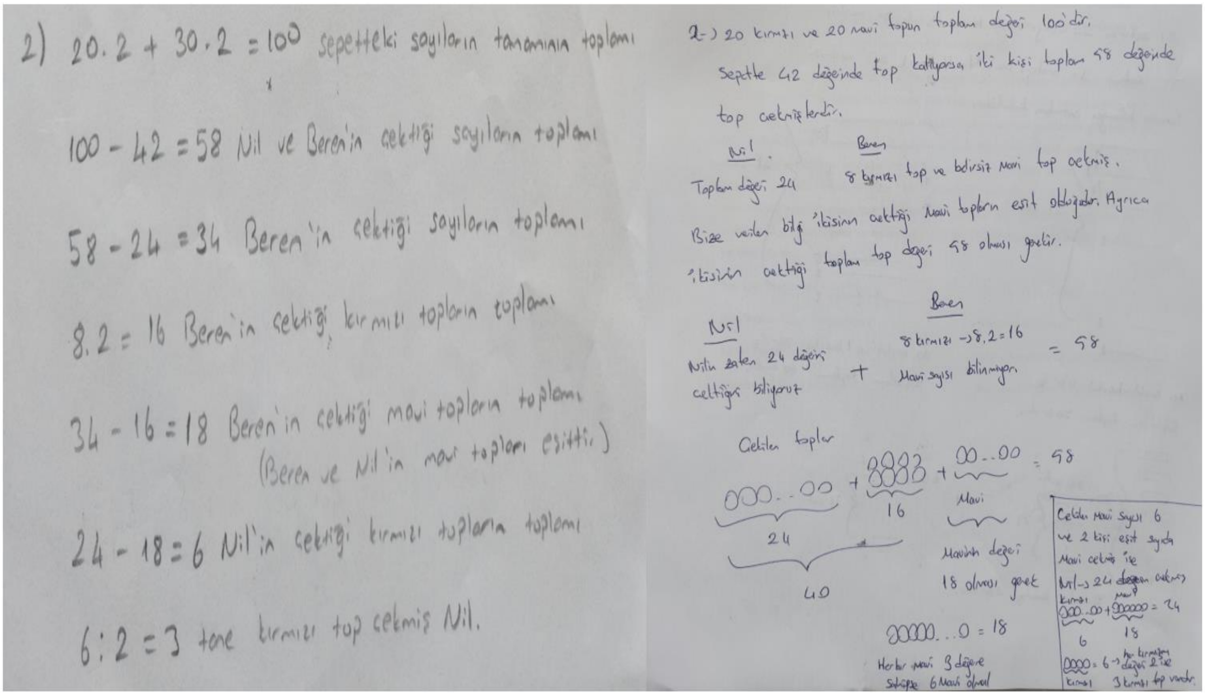

Figure 3. P_{18} *and* P_{32} *Answers to the Second Problem*

P_{18} found the sum of the numbers on the balls drawn by Nil and Beren after subtracting 42 from the sum of the numbers written on all the balls. Subtracted 24 and 16 (8 red) from this number and found the sum of the numbers in Beren's blue balls. Subtracted this number from 24 and found the sum of the numbers in the red balls. In the last step, the number of red balls was found by dividing by 2. Thus, P_{18} scorrectly solved the problem with the working backward strategy by working from the last to the first P_{32} found the correct answer by establishing a logical relationship between the color of the balls, the numerical value of each ball and the number of balls drawn by Nil and Beren. It is seen that both teachers were able to solve the second problem correctly by choosing different strategies without using variables.

15,1% of the participants solved the problem correctly by using variables. Most of them solved the problem by using symbols such as □, ∆, etc. instead of variables. Moreover, 9.1% of the participants could not solve the problem or solve the problem incorrectly despite using variables. The answers of the participants who solved the second problem correctly and incorrectly by representing the variable with various geometric symbols are presented in Figure 4.

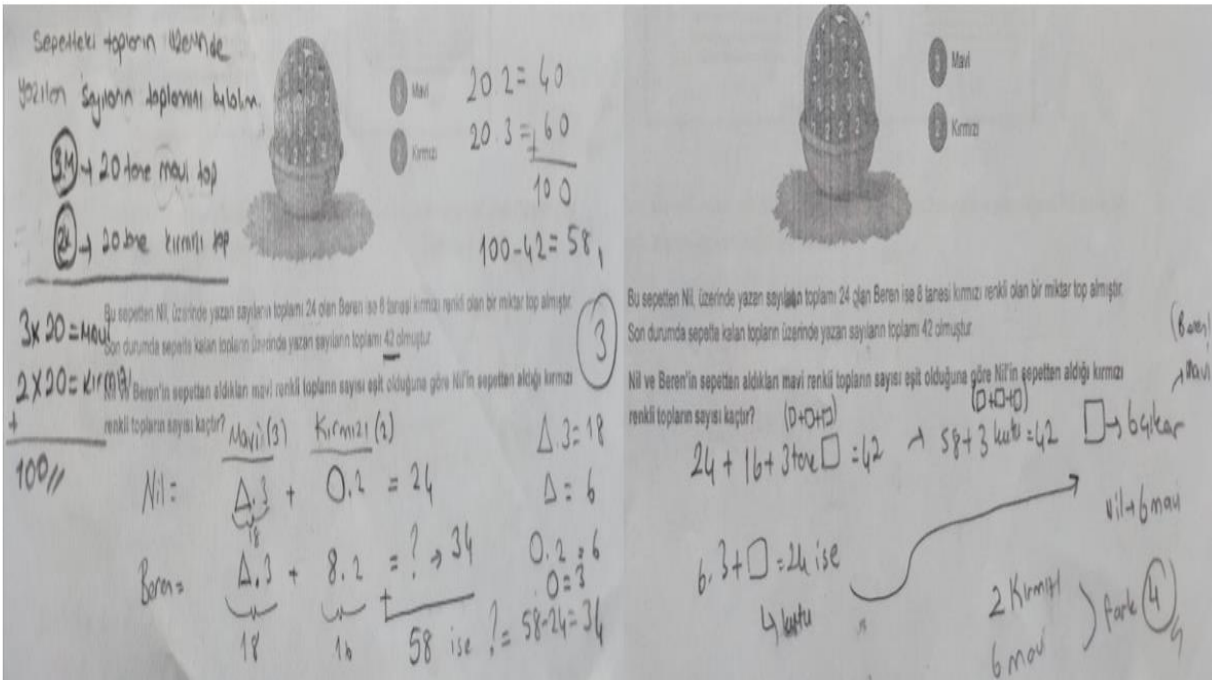

Figure 4. *P_{31} and P_{10} Answers to the Second Problem*

K31 assigned a variable by representing the number of red balls with ○ and the number of blue balls with ∆. The values of the variables were determined correctly by solving the equations (3.∆ +2.○= 24 and 3.∆ +8.2= 34). K10 also established the equation 24+16+3.□=42 by using the symbol □ instead of variable. However, it was observed that the problem could not be solved correctly because the equation was incorrect. Both teachers believe they have successfully solved the problem without using variables. The explanation of P_{31} supports this.

> *R: I saw that you used symbols in your solution. What was the reason for using symbols?*
>
> *P_{31}: I showed the problem's necessary value with a box. Then, I set up the equations and solved the problem.*

3.3. Findings regarding the Third Problem

Table 5 presents the distribution of the participants' answers to the third problem according to categories and codes.

Table 5. *Percentage and Frequency of the Categories and Codes related to the Third Problem*

Categories	Codes	f
Solving problem correctly (without using a variable)	Using unit fraction or ratio	11
	Using least common multiple	5
	Using models (area model, length model etc.)	4
Solving problem correctly (using a variable)	Using variables	1
	Representing the variable with a name	1
Incorrect solving (with or without using variables) or no solution	Solving incorrectly (without using a variable)	3
	Solving incorrectly (using a variable)	1
	No solution	7

More than half of the participants (60.6%) were able to solve the third problem without using variables. In addition, they mostly used the least common multiple and unit fraction or ratio strategies. The answers of the participants who solved the third problem without using variables are presented in Figure 5.

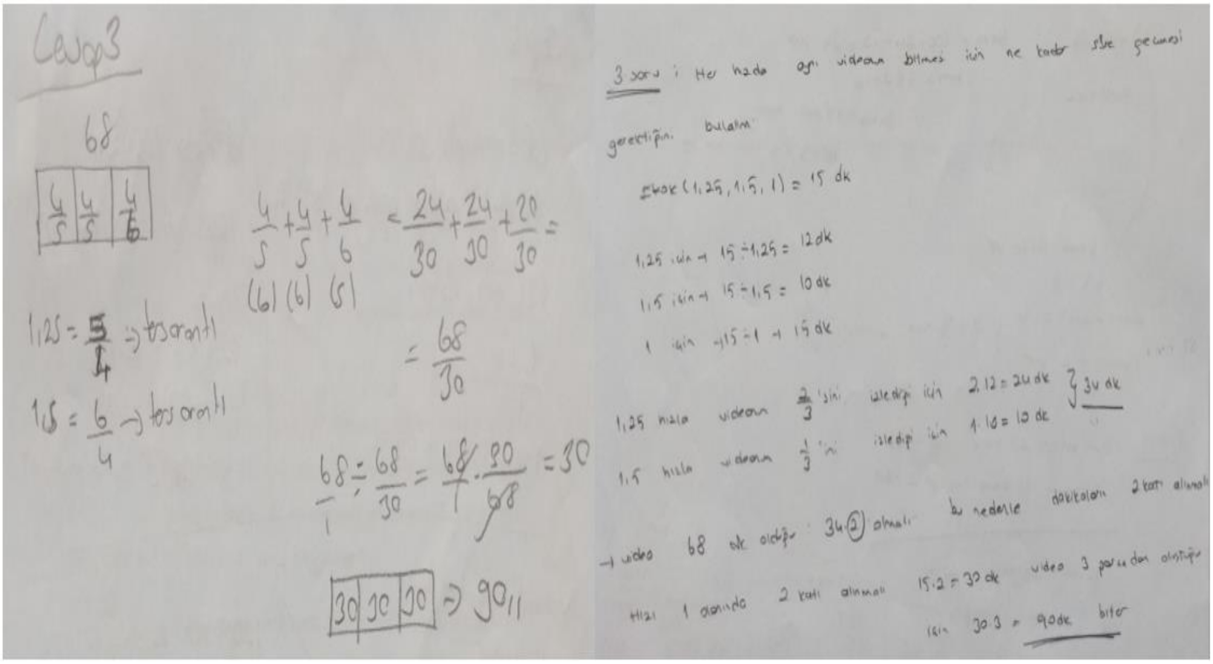

Figure 5. *P_{21} and P_{19} Answers to the Third Problem*

P_{21} expressed the total watching time in fraction format by establishing an inverse proportion between playing speed and watching time. The duration of the 1/3 video at normal speed was calculated and then multiplied by three to determine the total length of the video. The concepts of fraction and ratio-proportion were used to solve the problem. P19 found the least common multiple of the different playing speeds and determined the viewing times according to this value. Depending on the speed and the number of video parts, viewing times were found and compared with the value in the question. Finally, by multiplying the ratio values 2 and 15 (echo value), 1/3 of the video was found to be 30 minutes. The duration of the video, consisting of 3 parts, was calculated as 90 minutes. It is seen that both teachers were able to solve the third problem correctly by choosing different strategies without using variables.

In addition, 6% of the participants solved the problem correctly by using variables. Most of them solved the problem correctly by using symbols such as □, Δ instead of variables. Moreover, 33.3% of the participants solved the problem incorrectly or could not solve the problem despite using variables. The participants' answers, who solved the third problem correctly using variables and incorrectly without variables are presented in Figure 6.

P_{25} compared the playing speed specified in the problem and the average playing speed. The data obtained regarding the variable x were summed and equaled to the 68-minute duration given in the problem. The numerical value of the variable (x=2) was found and multiplied by the time at the normal playing speed to calculate the duration of the 1/3 video.

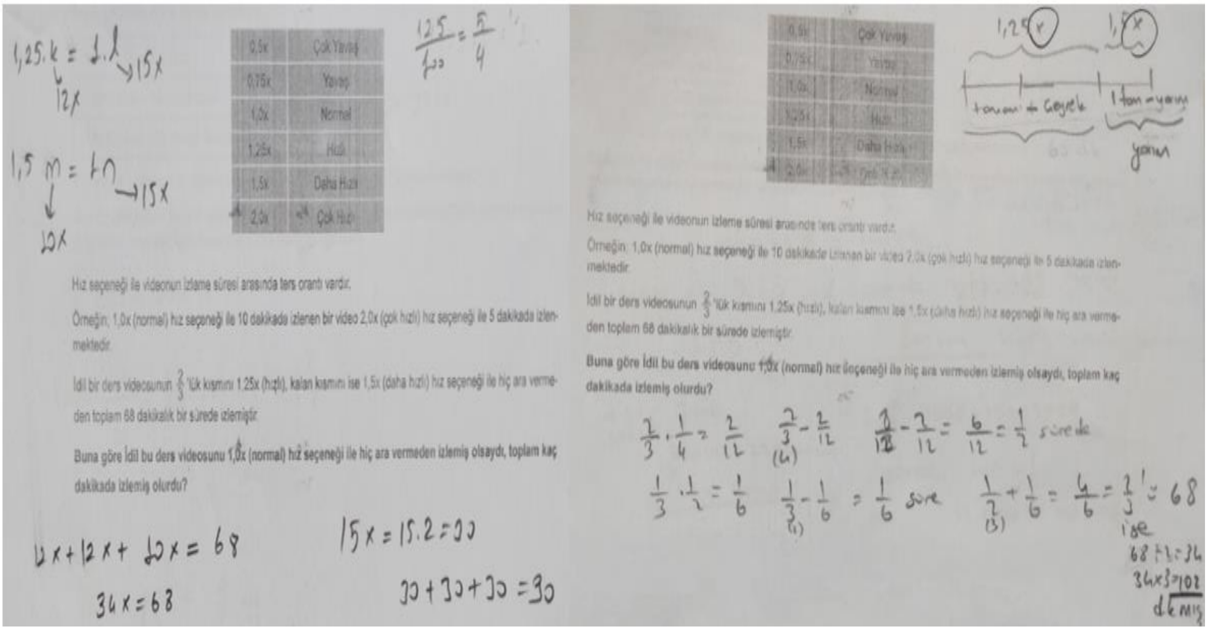

Figure 6. *P_{25} and P_2 Answers to the Third Problem*

P_2 found the differences between the specified playing speeds from the normal. These differences were multiplied by the 2/3 and 1/3 parts to which they belonged. The results were subtracted from the 2/3 and 1/3 parts of the video, and the early finishing times were found according to the normal playing speed and compared with the value 68 in the problem.

However, the real length of the video was calculated incorrectly. The interview dialogue about this is as follows.

R: Can you explain what you did to solve the problem?

P_2: I divided the video length into three parts. I tried to find the difference between the speeds given in the problem and the average speed and calculate how long it would take to finish early. I found a rate between the result I found and 68 minutes. I think I did it wrong...

R: Would it be easier for you if you used variables?

P_2: I think, yes. Because the problem is suitable for using variables

According to the dialogue above, it is understood that the participant had difficulty in solving the question without using variables.

3.4 Findings regarding the Fourth Problem

Table 6 presents the distribution of the participants' answers to the fourth problem according to categories and codes.

Table 6. *Percentage and Frequency of the Categories and Codes related to the Fourth Problem*

Categories	Codes	f (%)
Solving problems correctly (without using a variable)	Using models (area model, length model, etc.)	3
	Using a logical reasoning strategy	11
Solving problems correctly (using a variable)	Using variables	2
	Representing the variable with symbols (e.g. □, ∆)	6
	Representing the variable with a name	2
Incorrect solving (with or without using variables) or no solution	Solving incorrectly (without using a variable)	8
	No solution	1

Almost half of the participants (42.4%) were able to solve the fourth problem without using variables. In addition, they mostly used to use logical reasoning strategies. The answer of the participant who solved the fourth problem without using variables is given in Figure 7.

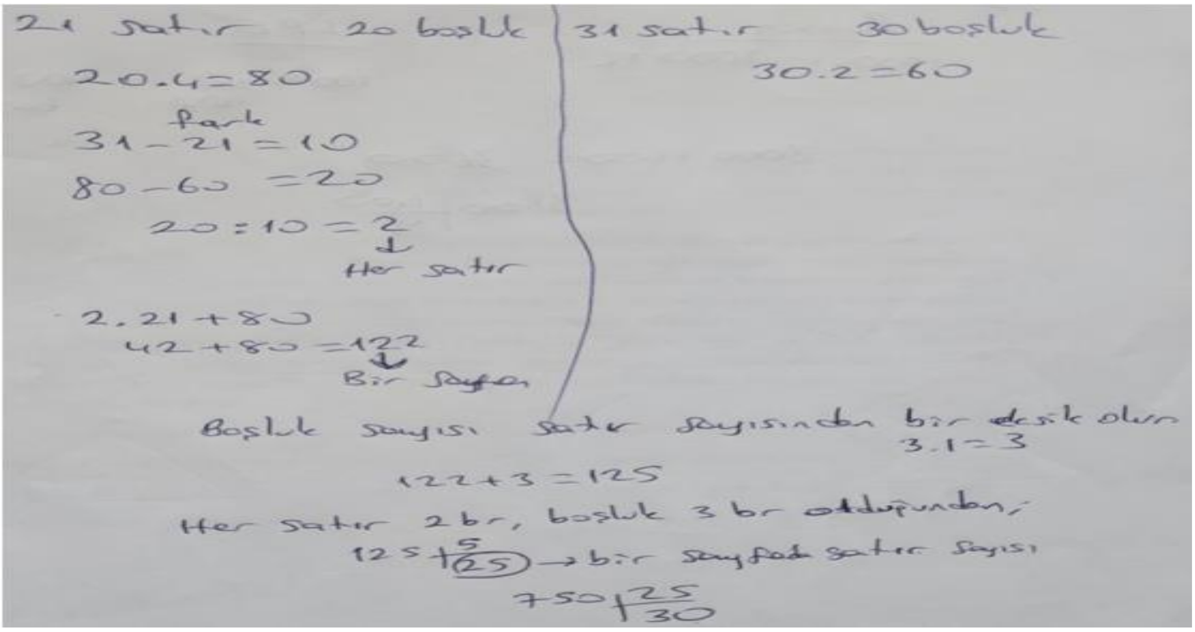

Figure 7. P_{17} *Answers to the Third Problem*

P_{17} calculated the length of a page as 122 according to different line spacing. The difference between the number of lines and line spacing was one, so an extra line spacing was added, and the number of lines and line spacing were equalized. Thus, the new page length was found to be 125. To find the number of lines that could fit on a page in the new situation, the page length was divided by the sum of the line lengths (2 units) and the line spacing (3 units). It was calculated that 25 lines fit on a page, and 30 pages are needed for a 750-page text in the problem.

30,4% of the participants solved the problem correctly by using variables. Most of them solved the problem correctly by using symbols such as □, ∆, etc. instead of variables. Moreover, 27.2% of the participants could not solve the problem or their solution without using variables was incorrect. The answers of the participants who solved the fourth problem correctly by representing the variable with a name and incorrectly without using a variable are presented in Figure 8.

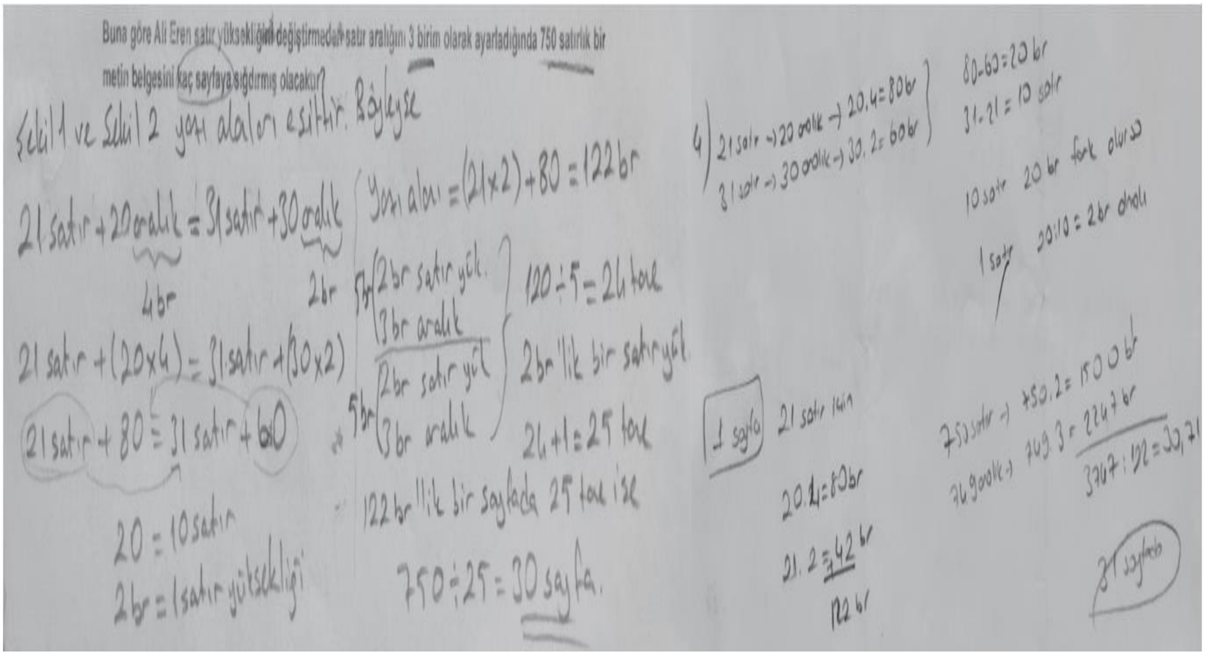

Figure 8. *P_{12} and P_3 Answers to the Third Problem*

P_{12} established an equation to express the page length that is equal in both cases according to different line spacing (21 lines + 20 spaces (4 units) = 31 lines + 30 spaces (2 units)). Names represent the row and spacing variables in the equation. After solving the equation and calculating the line height, the length of a page was found in units. Then, it was calculated that a 750-page text could fit on 30 pages for the condition that the line spacing was 3 units. P_3 found a line height and calculated a page length for the same page length by comparing different line spacing conditions. For the case where the line spacing was 3 units, the total heights of 750 lines and 749 lines spacing were found. The wrong solution was made by dividing by the page length. In this regard, some quotations from the interview with P_3 are presented.

R: I see that you established an equation using variables to solve the problem. Can you find a solution without using variables?

P_{12}: Actually, I did not use variables.

R: You used 'row' and 'range' instead of variables like x and y. Right?

P_{12}: Yes, I did not use them as variables.

As it can be understood from the participant's answers, the participant is not aware that the expressions row and interval are used as variables.

3.5. Findings regarding the Fifth Problem

Table 7 presents the distribution of the participants' answers to the fifth problem according to categories and codes.

Table 7. *Percentage and Frequency of the Categories and Codes related to the Fifth Problem*

Categories	Codes	f (%)
Solving problems correctly (without using a variable)	Using a logical reasoning strategy	8
	Make tables and lists	1
Incorrect solving (with or without using variables) or no solution	Solving incorrectly (without using a variable)	18
	No solution	6

27.2% of the participants solved the fifth problem without using variables. In addition, they mostly used to use logical reasoning strategies. The answers of the participants who solved the fifth problem without using variables are presented in Figure 9.

P_5 made a cost table according to fuel type and month variables and listed the costs of fuel types according to months. In the 7th month, LPG was costlier than gasoline, and in the 8th month, gasoline was costlier than LPG, so there were no cost calculations for the months after the 8th month. Therefore, in the arithmetic average of the values of the 7th and 8th months, it was seen that the costs were equalized, and the correct solution was made. P_{14} calculated the difference between monthly LPG and gasoline costs and found the monthly savings amount. Afterwards, the cost of installing the LPG system was divided by the amount of savings, and the number of months in which the costs were equal according to the fuel type was found, and the correct solution was made.

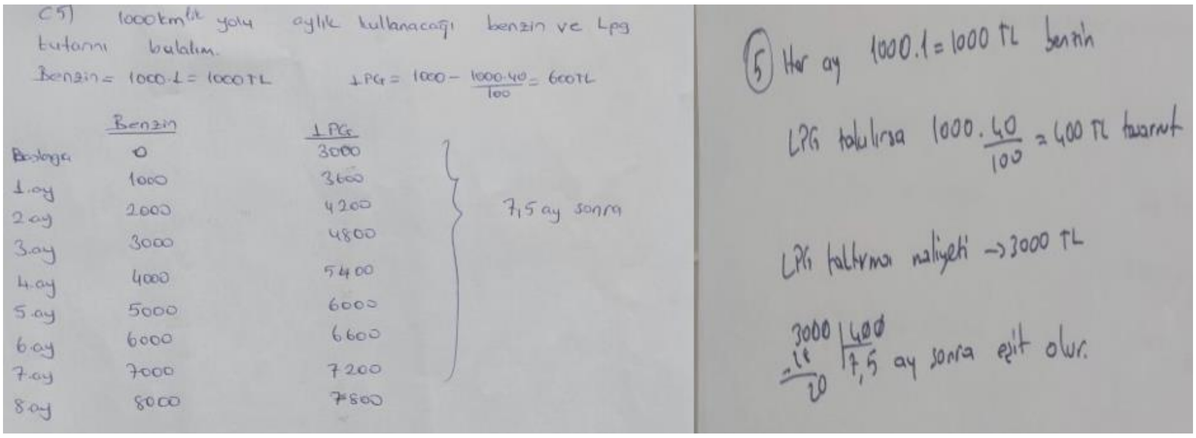

Figure 9. P_5 *and* P_{14} *Answers to the Third Problem*

More than half of the participants (72.8%) solved the problem incorrectly or could not solve it even though they used variables. Figure 10 presents the answers of the participants who solved the fifth problem correctly by representing the variable with a name and incorrectly without using a variable.

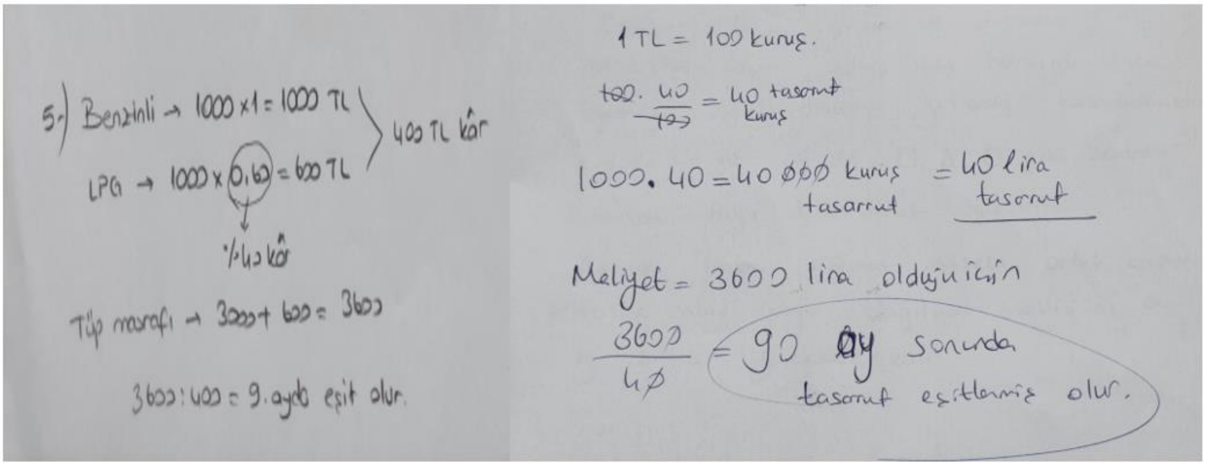

Figure 10. *P_{22} and P_{27} Answers to the Third Problem*

P_{22} calculated the monthly costs of gasoline and LPG fuel types and found the monthly savings amount. LPG system installation and servicing costs were summed and divided by the savings amount. Thus, it was found that the costs would be equalized according to fuel types in the 9th month. However, it was missed that the LPG servicing cost would not be included in the overall cost, and the incorrect solution was made. P_{27} made a mistake in converting a penny to Turkish lira and found the amount of savings incorrectly. Therefore, the problem was solved incorrectly.

3.6. General Trends

When all problems are considered, common strategies and errors are observed in students' problem-solving processes. Among the correct solutions, it is seen that especially the logical reasoning strategy is frequently used. This strategy is widely preferred among problems where correct solutions are produced without and without using variables (e.g., the second and fourth problems). This finding shows that students mostly rely on intuitive and analytical thinking skills. In addition, although systematic approaches such as making tables and lists are effective in some problems, this strategy is less preferred. In terms of incorrect solutions, it is observed that errors are more common in problems where variables are not used and that students have difficulty in producing solutions in such problems. Especially in the fifth problem, the high rate of incorrect solutions is striking, and this situation reveals that students have difficulty determining the correct strategies for some problems. In addition, when the "no solution" option is considered in all problems, it is seen that a particular group of students have difficulty even in starting to solve the problem. This may indicate fundamental deficiencies in problem-solving skills. These general trends synthesize the research findings and provide important insights into the strengths and weaknesses of students in their problem-solving processes. In particular, the prevalence of approaches based on logical reasoning strategies indicates that mathematical thinking skills need to be developed, while the frequency of wrong solutions and unsolvable situations indicates that students need support in specific strategic approaches.

4. DISCUSSION

The present study investigated elementary school mathematics teachers' ability to solve mathematical problems without using variables and the strategies they used. The results revealed that teachers' problem-solving skills without using variables were limited (Gökkurt Özdemir et al., 2018a). It also indicates that teachers prefer to solve mathematical problems with various strategies and that these strategies affect their problem-solving skills (Öçal et al., 2020). Different strategies lead to diversification in responses to problems. It shows that the strategies used in problem-solving processes are reflected in mathematical thinking skills. Therefore, the diversity of teachers' answers to problems implies that the effectiveness of the methods used in problem-solving processes reflects their mathematical thinking skills.

Teachers' skill level of solving problems correctly without using variables is 52% on average. They use guess and check, using models (area model, length model, etc.), using logical reasoning, making tables and lists, using unit fractions or ratios, and working backward strategies. This indicates that using variables is not always necessary and alternative strategies can be effective for certain problems. Although they find a solution without variables, it can be considered that teachers' solutions are inadequate for students at the concrete operations stage. The solution's lack of visual elements may make it difficult to understand the problem. Considering the importance of concrete elements and practical learning experiences within the scope of Piaget's theory of cognitive development, the lack of concretizing elements can be a problem, especially for students in the concrete operational stage (Börnert-Ringleb & Wilbert, 2018; Cerovac & Keane, 2024). It is known that visual elements such as figures and diagrams facilitate the understanding of the problem (Yavuz & Yüca, 2017). Therefore, in cases where teachers do not use visual support, more emphasis can be placed on students' experiences and understanding. In addition, students who use concretizing elements such as figures and diagrams are more successful in solving the problem (Chu, Rittle-Johnson & Fyfe, 2017; Xing, Corter & Zahner, 2016). While the students needed concrete support, the teachers preferred strategies such as logical inferences and using tables to find correct answers. It is essential that teachers use different strategies while solving the problem. It is seen that these strategies can be effective, especially in low-complexity problems, and correct answers can be obtained without using variables. This situation emphasizes the effect of teachers' awareness of not using variables and their strategies on the solving of problems.

Teachers' problem-solving strategies varied according to the nature of the problem and their level of mathematical thinking and knowledge. Some teachers thought that symbols were not variables and represented them in various ways. 19.4% of the participants solved the problem correctly by representing or using a variable. Symbols such as □, Δ or their names were used instead of variables. It was thought that they had solved the problem without using variables. However, it can be inferred that the difference between symbols and variables was not clearly understood (Cerovac & Keane, 2024). This may indicate teachers' lack of conceptual knowledge (Brown & Bergman, 2013). Moreover, some teachers were able to get incorrect results by using variables. This situation shows that teachers' awareness and knowledge of variables are insufficient. This shows that teachers do not know the concept of variables and need to develop more knowledge and awareness about the use of variables. It indicates that these teachers do not recognize the concept of variables, and they need to develop more knowledge and awareness about the use of variables (Gökkurt Özdemir, Koçak & Soylu, 2018b). Because it was seen that they used symbols as variables although symbols have the same function as variables. However, presenting the variable with labels or symbols different from x and y as usual does not mean it cannot be accepted as a variable. Accordingly, it is concluded that the teachers' knowledge of variables, one of the frequently used concepts in the problems, is insufficient. Therefore, teachers' knowledge and use of variables seem insufficient, and this situation is directly reflected in problem solving processes.

5. CONCLUSION AND LIMITATIONS

This study reveals that elementary school mathematics teachers use a wide range of strategies to solve problems without variables. Teachers' strategy choices directly affect their problem-solving abilities and mathematical thinking skills. The findings emphasize the need for a more comprehensive investigation of mathematical problem-solving strategies and variables used in teacher education. In particular, it is essential for teachers to develop their ability to use various problem-solving strategies effectively by the grade level as well as variable representation with symbols. In addition to the use of variables, it is concluded that teachers' knowledge and skills in problem-solving strategies need to be improved. Furthermore, it is significant for mathematics teachers to develop more effective strategies in problem-solving processes and to increase their awareness of the correct use of variables. In undergraduate education, courses on developing problem-solving strategies for students with different learning characteristics, along with the concept of variables, can be included. The findings suggest that there is a need for more research on teachers' use of variables and problem-solving strategies.

Although this study examined teachers’ ability to solve mathematical problems without using variables and the strategies they used in this process, there are some limitations. The strategies teachers use in problem-solving processes are limited only by the reported data. Investigations are needed to understand the cognitive processes behind these strategies fully. More detailed and longitudinal studies can be conducted to understand how teachers think during problem-solving and which factors affect strategy selection. In future research, in-depth analyses can be conducted on how teachers can improve their problem-solving strategies. In addition, it is essential to develop strategies for the difficulties teachers face in problem-solving processes and how these difficulties can be overcome. Although the study examines teachers’ problem solutions, it does not provide direct data on how students are affected by these strategies. Examining the effects of teachers’ strategy choices on students can be valuable, especially in understanding the role of these strategies on student achievement. Finally, a comprehensive assessment of teachers’ knowledge levels on variable use has not been conducted. A more in-depth examination of teachers’ lack of knowledge about the concept of variables can contribute to developing more effective educational programs to address these deficiencies.

6. CONFLICTS OF INTEREST

The authors declare no competing interests.

7. FUNDING

The authors did not receive any financial support.

8. ETHICS

This study was approved by the ethics committee of the Agri Ibrahim Çeçen University (*Reference number 83*), and each phase of the research was conducted according to the Declaration of Helsinki.

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